

Editor's note

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Hadron spectra and quarks

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INTRODUCTION

The search for an understanding of the nature of matter has repeatedly been aided by the idea that complexity in a system is evidence for a substructure of constituents that are small on the scale of the complex system. Thus it has been fruitful to progress from materials to atoms, from atoms to electrons and nuclei, from nuclei to nucleons, and from nucleons to quarks in the elucidation of the properties of matter. It is a direct consequence of quantum mechanics that probing physical systems on a decreasing scale requires increasing energies. In recent decades, the construction of ever higher energy accelerators has led to a remarkable progress in the understanding of what at this time appear to be the ultimate constituents of matter. This article describes the present state of understanding of these constituents, the *quarks*, their fundamental interactions, and the way in which they manifest themselves in the properties of nucleons and other subatomic particles.

Progress in the understanding of the physics of elementary particles has been made on two levels. On one side, major advances have followed from developments in quantum field theory. These include a better understanding of acceptable high-energy behavior in different theories, or, more technically, the problem of renormalizability; they include a better understanding of the difference between an underlying symmetry, such as manifests itself in the basic equations of motion (or, equivalently, the Lagrangian), and the apparently lower symmetry of the solutions, technically known as the problem of spontaneous symmetry breaking; finally, they include the notion of an energy-dependent coupling strength, which was necessary to understand why certain high-energy experiments could be interpreted in terms of particles consisting of almost free constituents, called partons, and identifiable with quarks. All of these developments involved an understanding of so-called non-Abelian gauge theories. On the other side, major quantitative advances in correlating detailed experimental information followed from taking the quark model of strongly interacting particles seriously, and applying the tools of symmetry, nonrelativistic quantum mechanics, relativistic corrections, and so on, to the model. These tools are much more familiar to most physicists than are the techniques of quantum field theory, and this is reflected in the coverage given to the two aspects in this tutorial article. Sections I, II, and V deal with the field theory domain. In Sec. I the elementary particles and the hierarchy of interactions is introduced. The notion of local gauge invariance as applied to a relativistic Lagrangian is discussed for the electromagnetic and weak interactions of leptons and quarks. In Sec. II the gauge theory that describes the strong interactions, quantum chromodynamics, is discussed, together with its implications for high-energy experiments, and other reasons for believing that it is the right theory. The topic of

fundamental interactions is again taken up in Sec. V, where present ideas about unifying all the interactions, and their experimental consequences, are discussed. In these sections the discussion is necessarily somewhat superficial; a monograph would be required to go into all the details of the derivations and calculations. The material is presented in spite of this difficulty because much of the discussion of the advances in this field is couched in terms of the fundamental theory, and ideas about unification are playing a role both in speculations about early cosmology, and in stimulating new experiments designed to test the stability of matter.

The parts of the recent advances that use the more familiar tools of particle physics are treated in Secs. III and IV. The first one of these deals with the spectroscopy of systems composed of light quarks, and Sec. IV deals with systems that contain one or more of the newly discovered heavy quarks. Here specific applications of symmetry are worked out in detail, transition amplitudes are calculated, and an attempt is made to develop some sort of intuition based on nonrelativistic quantum mechanics.

The bibliography consists of three parts. The first lists a few textbooks and monographs in which the subject matter of this review is treated in greater detail than is possible here. The second part lists a number of *Scientific American* articles that deal with the subject matter and in the third part a fairly comprehensive list of references to the original literature in this field is given.

I. ELECTROMAGNETIC AND WEAK INTERACTIONS OF LEPTONS AND QUARKS

The particles that have been identified in high-energy experiments fall into distinct classes. There are the *leptons* all of which have spin $\frac{1}{2}$ in units of \hbar . They may be charged or neutral. Charged leptons have electromagnetic and weak interactions; neutral ones only interact weakly. At this time three lepton pairs have been identified: the electron (e^-) and its associated neutrino (ν_e), the muon (μ^-) and the muon neutrino (ν_μ), and the tau (τ^-) with its neutrino (ν_τ). These particles all have antiparticles, in accordance with the predictions of relativistic quantum mechanics. There appear to exist "lepton type" conservation laws: the number of e^- plus the number of (ν_e), minus the number of the corresponding antiparticles e^+ and ($\bar{\nu}_e$), is conserved, and similarly for the muon- and tau-type leptons. These conservation laws are not associated with any deep underlying symmetry and may turn out to be approximate at some level. Table I lists the masses of the leptons.¹ The pattern of masses, the apparent masslessness of the neutrinos and the existence of three "generations" of lepton pairs are not understood.

Table I. Lepton masses and lifetimes.

Lepton	e^-	ν_e	μ^-	ν_μ	τ^-	ν_τ
Mass in MeV/c ²	0.511	$< 6 \times 10^{-5}$	105.66	< 0.57	1784	< 250
Lifetime in sec.	stable	stable	2.2×10^{-6}	stable	$< 2.3 \times 10^{-12}$	

In addition to leptons there exist *hadrons* that have strong interactions in addition to the electromagnetic and weak ones. There are hundreds of them. They have a variety of spins, both integral and half-integral, and their masses range from 135 MeV/c² for the π^0 to 10 545 MeV/c² for the recently discovered Y^m . The particles with odd half-integral spin are called *baryons*. There is clear evidence for baryon conservation: the number of baryons minus the number of antibaryons is conserved in all observed interactions. The best evidence for this is the stability of the lightest baryon, the proton. If the proton decays, it does so with a lifetime in excess of 10^{30} yr.² There is no deep principle that can be seen to be responsible for this conservation law, so it, too, like the lepton-type conservation law, may turn out to be approximate. The particles with baryon number $B = 0$ are called *mesons*. They have integral spin. The large number of hadrons, the ease with which they convert into each other, the "softness" of their interactions and the fact that they may be classified into multiplets that are very reminiscent of energy levels of atoms or molecules, all suggest that they are composite systems, in contrast to the leptons that at currently accessible energies give every indication of being pointlike.

The pointlike nature of the leptons is most clearly brought out by a study of their interactions. The successful theories that describe the electromagnetic and weak interactions of the leptons deal with the leptons as fundamental constituents. The application of quantum electrodynamics to new high-energy experiments carried out at the Stanford Linear Accelerator Center (SLAC) and at the Deutsches Elektronen Synchrotron (DESY) establishes that the electron size is less than $2-3 \times 10^{-3}$ fm.³ The theory of the electromagnetic interactions of charged leptons is a perfect model of a successful theory, and as such has guided the search for a fundamental description of the weak and strong interactions. For this reason it is useful to give a description of the theory at this stage.

Quantum electrodynamics

The only known way to describe local Lorentz invariant interactions is to construct a theory of fields that describe the leptons (the electron, for example) and the photons. The free lepton field, denoted by $l_\alpha(x)$, obeys the Dirac equation

$$i(\gamma^\mu)_{\alpha\beta} \frac{\partial l_\beta(x)}{\partial x^\mu} - m_0 l_\alpha(x) = 0, \tag{1.1}$$

where $l_\alpha(x)$ is a four-component spinor, whose indices will be suppressed in what follows. The four matrices $(\gamma^\mu)_{\alpha\beta}$ ($\mu = 0, 1, 2, 3$) are 4×4 matrices that satisfy the conditions¹⁰⁴

$$(\gamma^1)^2 = (\gamma^2)^2 = (\gamma^3)^2 = -(\gamma^0)^2 = -1,$$

$$\gamma^\mu \gamma^\nu = -\gamma^\nu \gamma^\mu \quad \mu \neq \nu.$$

With the introduction of $\bar{l}_\alpha(x) = l_\beta^+(x)(\gamma^0)_{\beta\alpha}$ one may write a Lorentz invariant Lagrangian density

$$\mathcal{L}_0 = i\bar{l}(x)\gamma^\mu \frac{\partial}{\partial x^\mu} l(x) - m_0 \bar{l}(x)l(x), \tag{1.2}$$

from which the Dirac equation follows as an equation of motion. The parameter m_0 is the *bare mass* of the lepton. This nomenclature serves as a reminder that interactions give rise to energy shifts, so that the true lepton mass m differs from m_0 . The Lagrangian density is invariant under the transformation

$$l(x) \rightarrow e^{i\lambda} l(x); \quad \bar{l}(x) \rightarrow \bar{l}(x) e^{-i\lambda}. \tag{1.3}$$

Such a transformation is called a global $U(1)$ transformation. It is global, because the transformation is the same at all space-time points x , and it is called $U(1)$ because it is the most general unitary transformation that transforms a field $l(x)$ into itself.

The consequences of this invariance are not trivial: as first shown by E. Noether,⁴ global invariances lead to conservation laws, and here the consequence is charge conservation. There is a current $j^\mu(x) = \bar{l}(x)\gamma^\mu l(x)$ that is conserved,

$$\frac{\partial}{\partial x^\mu} j^\mu(x) = 0, \tag{1.4}$$

and the charge

$$Q = \int d^3x j^0(x) = \int d^3x l^+(x)l(x) \tag{1.5}$$

is a constant of the motion. The Lagrangian density is not invariant under *local* $U(1)$ transformations, in which $\lambda = \lambda(x)$, because of the derivative in the Lagrangian. It turns out that the requirement of local invariance can only be met if one introduces an additional field whose own transformation properties compensate for the term $\partial\lambda(x)/\partial x^\mu$ that appears from applying the derivative to the transformed field $e^{i\lambda(x)}l(x)$. Invariance is maintained if the Lagrangian is

$$\mathcal{L}_e(x) = i\bar{l}(x)\gamma^\mu \left(\frac{\partial}{\partial x^\mu} - ie_0 A_\mu(x) \right) l(x) - m_0 \bar{l}(x)l(x), \tag{1.6}$$

and the transformations are

$$\begin{aligned} l(x) &\rightarrow e^{i\lambda(x)} l(x), \\ \bar{l}(x) &\rightarrow \bar{l}(x) e^{-i\lambda(x)}, \\ A_\mu(x) &\rightarrow A_\mu(x) - \frac{1}{e_0} \frac{\partial \lambda(x)}{\partial x^\mu}. \end{aligned} \tag{1.7}$$

The replacement of a gradient ∇ by $[\nabla - ie_0 \mathbf{A}(x)]$ is familiar from nonrelativistic quantum mechanics, and gives rise to what is called minimal coupling. The local transformation is called a gauge transformation, and $A_\mu(x)$ is called the

Table II. Comparison between theory and experiment in quantum electrodynamics. ^a

Physical quantity	Theory	Experiment
Fine structure constant	input	$[137.035\,987\,(29)]^{-1}$
Photon mass	0	$< 1.2 \times 10^{-21} m_e$
Deviation from gyromagnetic ratio		
$a = (g - 2)/2$		
$a_e \times 10^{12}$	1 159 652 570 (150)	1 159 652 200 (40)
$a_\mu \times 10^9$	1 165 921 (8.3)	1 165 924 (8.5)
Muonium hyperfine structure in kHz	4 463 297.9 (3.0)	4 463 302.35 (.52)
Positronium fine structure in MHz	203 400.3	203 384.9 (1.2) 203 387.0 (1.6)

^a The numbers are taken from T. Kinoshita (Ref. 5); S. J. Brodsky and S. D. Drell, Phys. Rev. D 22, 2236 (1980); J. Bailey, Nucl. Phys. B 150, 1 (1979); F. Farley and E. Picasso, Ann. Rev. Nucl. Part. Sci. 29, 243 (1979).

gauge field, or vector potential. If that field is to have a dynamical role, that is, have equations of motion (Maxwell's equations), a term quadratic in a derivative of $A_\mu(x)$ must appear in the Lagrangian. The only term that is quadratic and gauge invariant at the same time is of the form

$$\mathcal{L}_A = -\frac{1}{4}[F_{\mu\nu}(x)]^2, \quad F_{\mu\nu} = \frac{\partial A_\nu(x)}{\partial x^\mu} - \frac{\partial A_\mu(x)}{\partial x^\nu}. \quad (1.8)$$

This is just the usual $\mathbf{E}(x)^2 - \mathbf{B}(x)^2$ of electrodynamics. Notice that a term like

$$\mathcal{L}'_A = -\frac{1}{2}\mu_0^2[A_\mu(x)]^2, \quad (1.9)$$

which represents a photon mass, violates local gauge invariance. Gauge fields represent massless particles. A very attractive feature of the principle of local gauge invariance is that it dictates a unique form of the interaction between photons and leptons¹⁰⁵

$$\mathcal{L}_i - \mathcal{L}_0 = e_0 \bar{l}(x) \gamma^\mu l(x) A_\mu(x). \quad (1.10)$$

The theory described by (1.6) and (1.8) is called quantum electrodynamics. When account is taken of the fact that the fields appearing here are really quantum fields, the theory is in extremely good agreement with experiment, as is illustrated in Table II.⁵ It should be stressed that gauge invariance determines the coupling of any charged particle to the radiation field.

In working out the consequences of the equations of motion of quantum electrodynamics one uses perturbation theory to calculate scattering amplitudes. The technique commonly used is the one devised by R. P. Feynman,⁶ in which the perturbation series is put in a one-to-one correspondence with graphs, or *Feynman diagrams*, that are put together out of propagators and vertices involving particles that appear in the Lagrangian (Fig. 1), and the diagrams lead from a well-defined initial to a well-defined final state.

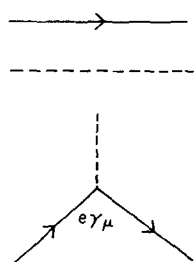


Fig. 1. Solid line represents electron propagator, dashed line represents photon propagator, and the vertex represents the electron-photon interaction described by (1.10).

Figure 2 illustrates electron-electron scattering (a) to lowest order with the minimum of two vertices, and (b) — (d) to order e^4 . The internal lines (propagators) transfer four-momentum only constrained by the conservation law at the vertices, and thus describe virtual particles, for which $E^2 - p^2 \neq m_e^2$, and which are therefore "off the mass shell." A diagram like (b) describes a self-energy contribution; the diagram (c) describes a modification of the radiation field caused by the possibility of creating a virtual electron-positron pair in the vacuum, and is called a vacuum polarization term.

The difficulty caused by infinite contributions of closed loops in which unconstrained four-momenta that are carried around the loop (Fig. 3) are summed over was solved by Tomonaga *et al.* in the late 1940s.⁷ To make sense of the infinite integrals, the momentum integration must be cut off at some upper limit. For a class of theories, that includes

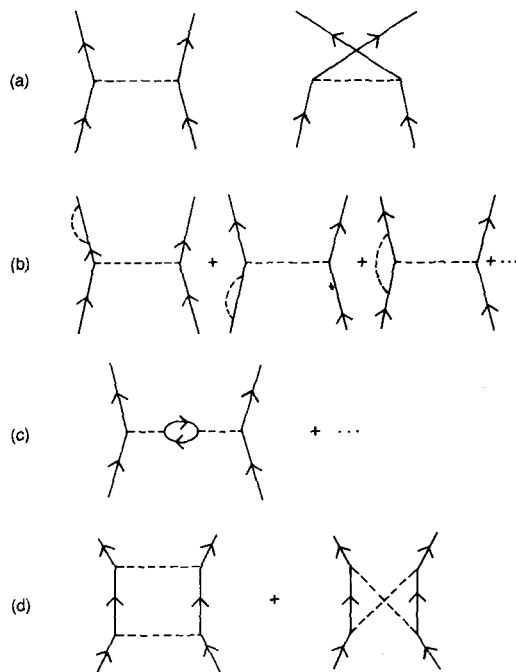


Fig. 2. Feynman diagrams for electron-electron scattering. (a) Graphs contributing to order e^2 in the amplitude; (b) order e^4 corrections due to electron interacting with the self-field; (c) order e^4 corrections due to polarization of the vacuum; (d) order e^4 corrections due to double-photon exchange.

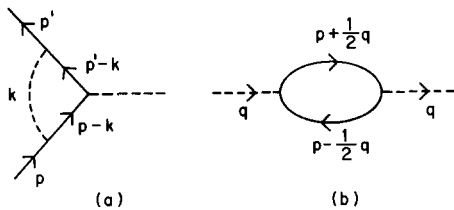


Fig. 3. Closed-loop integrals: in the vertex correction (a) k is an unconstrained four-momentum; in the vacuum polarization (b) p is an unconstrained four-momentum. The evaluation of the amplitudes involves integration over all values of these momenta.

quantum electrodynamics, the effect of the cutoff on physical amplitudes can be gathered into *renormalizations* that change the unobservable bare mass into the physical mass ($m_0 \rightarrow m$), and the bare charge into the physical charge. After this is done, and the fields are rescaled, the cutoff dependence vanishes in the remainder of the amplitudes, when the cutoff is allowed to go to infinity. The cutoff procedure must respect the symmetries, that is, Lorentz invariance and gauge invariance. The resulting finite amplitudes appear as a series in the dimensionless parameter $\alpha = e^2/4\pi\hbar c = 1/137$, whose smallness makes the series converge rapidly, provided the photon momentum squared, q^2 , at the vertices, is not too large. The reason for this qualification is that because of vacuum polarization the expansion is not really in terms of α^n , but rather in terms of $\alpha^n(\log q^2)^m$ and the convergence of such a series is much poorer. The reason for this behavior is that a charge in the vacuum “polarizes” the vacuum by creating virtual electron-positron pairs, thus shielding the original charge. The observed charge is reduced. At short distances, corresponding to large q^2 , the unscreened “bare” charge is seen, and this is larger. Effectively the summing up of leading vacuum polarization effects as a series in $\alpha \log q^2$ leads to a momentum-dependent coupling $\alpha(q^2)$, which reduces to $1/137$ for $q^2 \rightarrow 0$. At values of $q^2 = m^2 \exp(1/\alpha)$ the coupling becomes very strong. The summing up of the series in $\log q^2$ is no longer justified, and one does not really know what the ultrahigh-energy behavior of quantum electrodynamics is like. This is of no practical significance, but we shall see in Sec. V that the growth of the momentum-dependent coupling for quantum electrodynamics has some significance.

The exciting development of the last decade has been the realization that the gauge principle can be used to construct an equally successful theory of weak interactions that bears a strong resemblance to quantum electrodynamics, and that the theories can be meshed together in a theory of the electroweak interactions.

Weak interactions

In contrast to the electromagnetic interactions, whose form was already contained in classical electrodynamics, it took many decades of experimental and theoretical work to discover the compact phenomenological form of the interaction Lagrangian. For the interactions studied until the early 1970s,

$$\mathcal{L}_W = (1/\sqrt{2})G J_\alpha^+(x) J^\alpha(x) \quad (1.11)$$

provided a complete description, except for the phenomenon of the violation of time-reversal invariance discovered by J. Cronin and V. Fitch and co-workers,⁸ which was believed to be a manifestation of some kind of superweak

interaction. The currents appearing in (1.11) have the form

$$J^\alpha(x) = J_{\text{lepton}}^\alpha + J_{\text{hadron}}^\alpha \quad (1.12)$$

The leptonic current is of the form

$$J_{\text{lepton}}^\alpha = \sum_{\text{leptons}} \bar{l}(x) \gamma^\alpha (1 - \gamma_5) \nu_l(x). \quad (1.13)$$

Here γ_5 is a 4×4 matrix constructed out of the Dirac matrices, $\gamma_5 = \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, which has the property that $(\gamma_5)^2 = 1$ and $\gamma^\mu\gamma_5 = -\gamma_5\gamma^\mu$ for all μ . The form of the current differs from that appearing in (1.4) in that the two particles appearing in it have different charges. The charge of the particle entering the vertex, ν_l , is one unit larger than that of the particle leaving the vertex, l , and the current is thus called a *charged current*. The presence of both polar (γ^α) and axial ($\gamma^\alpha\gamma_5$) vector terms in J^α leads to cross terms that violate parity. The coupling G is not dimensionless, but if we write

$$2\sqrt{2}G = e^2/M_W^2 \quad (1.14)$$

then $M_W = 37.3 \text{ GeV}/c^2$. Numerically the interaction is very weak, and all observed phenomena in the leptonic weak interactions are accurately described by lowest-order graphs involving a four-lepton vertex (Fig. 4). Higher-order graphs involve higher powers of M_W in the denominator, and hence higher powers of the momentum in the numerator. This leads to more and more divergent terms in successive orders of perturbation theory, and the theory is not renormalizable.¹⁰⁶

In spite of the vast phenomenological differences with quantum electrodynamics, the presence of vector currents suggested a structure analogous to that theory, with a massive vector field [massive, to explain the pointlike character of the effective coupling (1.11)] carrying electric charge, mediating the weak interactions. Such theories are renormalizable only under very special circumstances, and that is when there is gauge invariance.

The generalization of the $U(1)$ transformations considered before involves a unitary transformation that mixes a set of fermion fields. The transformation

$$\begin{aligned} |\psi_A(x) &\rightarrow U_{AB} \psi_B(x), \\ \bar{\psi}_A(x) &\rightarrow \bar{\psi}_B(x) U_{BA}^+, \\ UU^+ &= U^+U = 1, \end{aligned} \quad (1.15)$$

on n fermion fields involves $n \times n$ unitary matrices that may be chosen to have determinant 1. The set of all such matrices forms a group. These matrices may be parametrized by $(n^2 - 1)$ “phases” α_i in the form

$$U_{AB} = (e^{iT\alpha_i})_{AB}. \quad (1.16)$$

The matrices $(T_i)_{AB}$ form elements of an algebra, and they must satisfy certain commutation relations that take the form

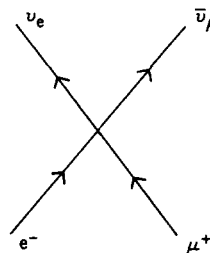


Fig. 4. Basic vertex for four-lepton weak interaction described by (1.11).

$$[T_i, T_j] = i f_{ijk} T_k. \quad (1.17)$$

The determinantal condition implies that the matrices T_i are traceless. They are Hermitian and $n \times n$, and thus the number of independent phases is $n^2 - 1$. The f_{ijk} ($i, j, k = 1, \dots, n^2 - 1$) are called structure constants. When the phases α_i are local $[\alpha_i(x)]$, we have a generalization of the gauge transformation (1.7). The requirement that the theory be invariant under a local $SU(n)$ (as the above transformation law is called) leads to the introduction of an analog of the vector potential $A_\mu(x)$. The generalization is an $n \times n$ Hermitian matrix $[W_\mu(x)]_{AB}$ and it appears in combination with the derivative in the same "minimal coupling" form that $A_\mu(x)$ appears in (1.6). The Lagrangian density

$$\mathcal{L} = i\bar{\psi}_A(x)\gamma^\mu\left(\delta_{AB}\frac{\partial}{\partial x^\mu} - igW_\mu(x)_{AB}\right)\psi_B(x) \quad (1.18)$$

is locally invariant provided that the matrix W_μ transforms according to

$$W_\mu(x) \rightarrow U(x)W_\mu(x)U^\dagger(x) + \frac{i}{g}U(x)\frac{\partial}{\partial x^\mu}U^\dagger(x). \quad (1.19)$$

There are $(n^2 - 1)$ gauge fields W_μ^i that form the elements of the matrix $W_\mu(x)$. We can define them by

$$W_\mu(x)_{AB} = W_\mu^i(x)(T_i)_{AB}. \quad (1.20)$$

The combination $[\partial/\partial x^\mu - igW_\mu(x)]$ is called the covariant derivative.

The transformation law (1.19) is very similar to (1.7), but the first term shows that the gauge field itself transforms under the group, in a manner that is similar to the product $\psi_A\psi_B$ ($A \neq B$). Thus the vector field carries the quantum numbers that are carried by the fermion fields. This is just what is needed, since the vector field mediating a charged current transition $\nu_e \rightarrow l^-$ must carry off the conserved electric charge. This complication manifests itself in the form of the analog of the electromagnetic field $F_{\mu\nu}$. Here it has the form⁹

$$F_{\mu\nu}^i(x) = \frac{\partial}{\partial x^\mu}W_\nu^i - \frac{\partial}{\partial x^\nu}W_\mu^i + g f_{ijk}W_\mu^jW_\nu^k, \quad (1.21)$$

and it transforms according to $F_{\mu\nu}^i T_i \rightarrow U F_{\mu\nu}^i T_i U^\dagger$. Thus the kinetic energy term for the vector field

$$\mathcal{L}_W = -\frac{1}{4}(F_{\mu\nu}^i)^2 \quad (1.22)$$

is invariant. It includes self-interaction terms that are trilinear and quadrilinear in the field, giving rise to the vertices shown in Fig. 5. The proof that such a theory could be relevant to the weak interactions had to await the solution of the vector mass problem. Gauge invariance demands masslessness, whereas the phenomenology demands a very massive W boson. The discovery of the mechanism that generates a W mass without loss of renormalizability¹⁰ is one of the major developments of modern quantum field theory.

A theory can have a symmetry and yet its ground state may break that symmetry. The standard example is an infinite ferromagnet at 0 K, which has as its underlying theory the rotationally invariant Coulomb interaction, but which has a definite direction associated with it. The ground state is infinitely degenerate in that all directions represent equally good ground states. Transitions between these

ground states are not possible, because for an infinite ferromagnet an infinite amount of energy is required to make a transition. Low-lying excitations in the ferromagnet involve spin waves, which consist of long-wavelength patterns of spins deviating from the original direction. The wavelengths can be arbitrarily long and the energy-momentum relation for these excitations is that of a massless particle. In particle physics a symmetry may be realized in an analogous way,¹¹ with massless bosons representing the low-lying excitations. These so-called Nambu-Goldstone bosons have the amazing property that in the presence of long-range forces they disappear.¹² Gauge theories yield long-range forces, and there the bosons become incorporated into the vector fields: these, when they are massless, have only two spin components (two states of transverse polarization) and the Goldstone boson becomes the third component necessary for a massive particle of spin 1, while at the same time "shielding" the long-range force, giving it a finite range, equivalent to giving the vector particle a mass. This mechanism, the Higgs mechanism, named after one of its discoverers, has its counterpart in superconductivity in the Meissner effect, which describes the attenuation of an external magnetic field inside a superconductor beyond a surface layer of finite range, the penetration depth.

S. Weinberg¹³ and A. Salam¹⁴ made use of this very beautiful development to construct a theory of the weak and electromagnetic interactions based on the group structure $SU(2) \times U(1)$ suggested some years earlier by S. L. Glashow.¹⁵ $SU(2)$ contains two charged ($Q = \pm 1$) vector meson fields W^\pm that may be chosen to couple to the charged currents that induce transitions between the neutrino and the electron, for example. The remaining two neutral vector mesons lead to the weak neutral transitions and to the electromagnetic ones. There is a complication that stems from the fact that the neutrino appears in the form of its "left-handed" projection

$$\nu_{eL} = \frac{1}{2}(1 - \gamma_5)\nu_e. \quad (1.23)$$

Thus the $SU(2)$ partner of the ν_{eL} must be $e_L = \frac{1}{2}(1 - \gamma_5)e$. Whereas the neutrino is massless, the electron is not, and thus the $e_R = \frac{1}{2}(1 + \gamma_5)e$ component is present. Thus the doublet (ν_{eL}, e_L) belongs to a two-dimensional representation of $SU(2)$ while e_R is a singlet under $SU(2)$. The electron mass term

$$m_0\bar{e}e = m_0(\bar{e}_L e_R + \bar{e}_R e_L)$$

transforms like a doublet, and cannot therefore appear in a symmetric Lagrangian. It must emerge through the same kind of spontaneous symmetry breaking that gives rise to the mass of the W . To assign the quantum numbers of $U(1)$ to the fields we use the relation

$$Q = T_3 + \frac{1}{2}Y, \quad (1.24)$$

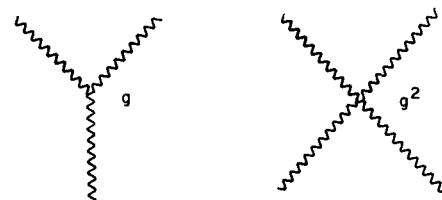


Fig. 5. Three- and four-vector-boson vertices contained in the gauge field term in the Lagrangian (1.22).

where Y is the "hypercharge" of $U(1)$ and T_3 is the third component of the "weak isospin" $SU(2)$. Thus the L components have $Y = -1$ and the R component has $Y = -2$. With these assignments, gauge invariance dictates unique couplings. The Lagrangian, in addition to the pure vector meson terms, is

$$\begin{aligned} \mathcal{L} = & (\bar{\nu}_{eL}, \bar{e}_L) \left(i\gamma^\mu \frac{\partial}{\partial x^\mu} + g(\frac{1}{2}\tau^\alpha)\gamma^\mu W_\mu^\alpha \right. \\ & \left. + \frac{1}{2}g'(-1)\gamma^\mu B_\mu \right) \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} + \bar{e}_R \left(i\gamma^\mu \frac{\partial}{\partial x^\mu} \right. \\ & \left. + \frac{1}{2}g'(-2)\gamma^\mu B_\mu \right) e_R. \end{aligned} \quad (1.25)$$

The Pauli matrices $\tau^\alpha/2$ are the $SU(2)$ matrices, and the coupling is g . The $U(1)$ coupling is conventionally taken to be $\frac{1}{2}g'$. A rearrangement of the neutral terms yields the electromagnetic coupling

$$e(\bar{e}\gamma^\mu e)A_\mu \quad (1.26)$$

provided the definition

$$A_\mu \equiv \frac{gB_\mu + g'W_\mu^3}{(g^2 + g'^2)^{1/2}}, \quad e = \frac{gg'}{(g^2 + g'^2)^{1/2}} \quad (1.27)$$

is used. The orthogonal linear combination, labeled Z^0 , appears in the form

$$\frac{eZ_\mu^0}{\sin\theta_w \cos\theta_w} \left[\sin^2\theta_w \bar{e}\gamma^\mu e + (\nu_{eL}, e_L)\gamma^\mu \frac{\tau^3}{2} \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \right], \quad (1.28)$$

where the parameter

$$\sin^2\theta_w \equiv g'^2/(g^2 + g'^2) = 0.215 \pm 0.012 \quad (1.29)$$

has the experimental value listed above. The coupling in (1.28) leads to weak interactions in which there is no charge transfer. These are the so-called neutral current interactions, leading to reactions such as

$$\begin{aligned} \nu_\mu + e^- & \rightarrow \nu_\mu + e^-, \\ \nu_\mu + \text{hadron} & \rightarrow \nu_\mu + \text{hadrons}, \end{aligned}$$

as well as parity-violating electromagnetic-weak interference effects in primarily electromagnetic processes, such as the scattering of polarized electrons by deuterons and other nuclei. Neutral current effects were first discovered at CERN in 1972 and are being measured with ever increasing precision. The data are in excellent agreement with experiment when the value of the mixing angle in (1.29) is used.

The spontaneous symmetry breaking could be a dynamical effect, as it is for the ferromagnet. In the Weinberg-Salam model it is put in by hand, by including in the theory an $SU(2)$ doublet of scalar mesons with $Y = +1$, coupling to the vector bosons in a gauge invariant way. Symmetry breaking is obtained by writing

$$\begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix} = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) + v \end{pmatrix}, \quad (1.30)$$

where $v = \langle 0|\phi^0(x)|0\rangle$. The nonvanishing of v implies that the vacuum is no longer a gauge invariant state. Further details have no place in this brief review.¹⁶ Suffice it to say that (i) renormalizability survives spontaneous symmetry breaking¹⁰; (ii) of the four scalar meson fields, three give masses to the vector bosons, and the fourth one remains a particle, the Higgs meson, in this form of the theory. When radiative corrections are included, the masses of the vector

bosons, with the above value of the mixing angle, are predicted to be¹⁷

$$\begin{aligned} m_w &= 83.0 \pm 2.4 \text{ GeV}/c^2, \\ m_{z_0} &= 93.8 \pm 2.0 \text{ GeV}/c^2. \end{aligned} \quad (1.31)$$

The reason for going into so much detail on this subject is that gauge theories also appear to describe the strong interactions, and that the mechanism of the weak interactions of hadrons is fundamentally the same as that of the leptons. Furthermore, the possibility of a unification of all of the interactions requires some understanding of the electro-weak interaction.

Hadrons and quarks

Hadrons are clearly composite objects. They come in large families. Central to the emergence of a classification of these families was the recognition of internal symmetries, which were first observed in the near identity of the proton and the neutron.¹⁸ These were described as doublets under an internal $SU(2)$ symmetry, called the I -spin symmetry. The pions, believed to be the principal carriers of the nuclear force, come in three charge states, π^+ , π^0 , and π^- , and their near identity makes it natural to assign these particles to an I -spin triplet ($I = 1$). The first highly unstable hadronic system, found as a resonance in pion-nucleon scattering, was soon identified to be an $I = 3/2$ state. In the 1960s very many such states were discovered and classified. A series of new "strange" particles discovered in the 1950s could further be classified by assigning to them a new quantum number, "strangeness,"¹⁹ and it took another eight years before M. Gell-Mann²⁰ and Y. Ne'eman²¹ proposed a unification of I -spin symmetry and strangeness into an approximate internal $SU(3)$ symmetry. The lowest-lying mesons belonged to an octet, as did the lowest-lying baryons, while the states containing the $I = 3/2$ resonance belonged to a ten-dimensional representation of $SU(3)$. These brief remarks cannot convey in any way the confusion that was cleared up, first by the discovery of strangeness, and then by the discovery of $SU(3)$. Large numbers of new particles were predicted and discovered, resonance widths could be correlated, as could masses within a multiplet, once a simple $SU(3)$ assignment of the symmetry breaking was proposed. The hadronic weak interaction vector currents were identified by Feynman and Gell-Mann as the I -spin currents,²² and this was extended by N. Cabibbo to the hadronic currents responsible for strangeness-changing weak decays.²³ Gell-Mann enlarged the $SU(3)$ symmetry to *chiral symmetry* $SU(3)_L \times SU(3)_R$ within which the hadronic axial weak currents found a place as natural as that of the vector currents.²⁴ Independently of work on symmetry, attempts to understand the many particles as dynamic consequences of certain patterns of exchange of particles, in which the unitarity relations of scattering amplitudes and their analyticity properties played a crucial role, led to many interesting insights, and a very thorough understanding of low-energy particle physics. The most interesting result of this program, led by G. F. Chew,²⁵ was a classification scheme in which particles of a given internal symmetry multiplet but different spins were assigned to families named *Regge trajectories*²⁶ that joined together particles in sequences such as $(0^+, 1^-, 2^+, \dots)$ and $(0^-, 1^+, 2^-, \dots)$. Some simple assumptions led to the spin-mass connection

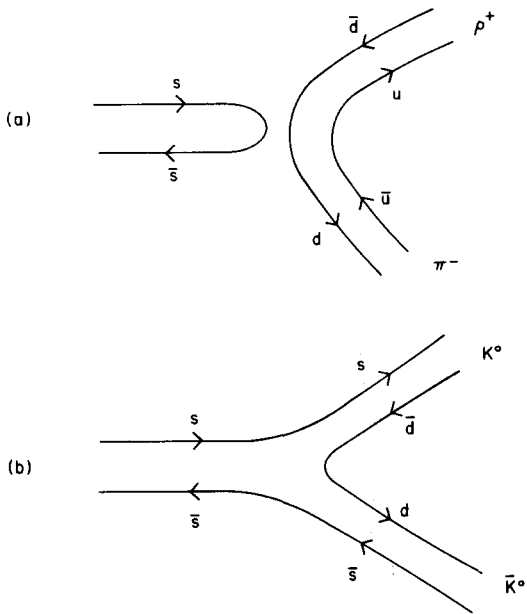


Fig. 6. (a) Decay $\phi \rightarrow \rho^+ \pi^-$ requires a quarkless intermediate state; (b) the decay $\phi \rightarrow K\bar{K}$ can proceed without such a gap.

$$J = \alpha_0 + \alpha' M^2, \quad (1.32)$$

The large number of multiplets, the fact that all mesons appear to belong to the **1** and **8** representations of $SU(3)$ and all baryons to the **1**, **8**, or **10**, and the absence of the fundamental **3** and $\bar{\mathbf{3}}$ representations were puzzling. M. Gell-Mann²⁷ and G. Zweig²⁸ solved the puzzle by proposing that hadrons be composite particles constructed out of $SU(3)$ triplets. They introduced a fundamental triplet, the *quark*,²⁹ and proposed that all the integral spin mesons be made of quarks and antiquarks, while the odd half-integral spin baryons be made of three quarks. The quarks were assumed to have spin $\frac{1}{2}$ and form an $SU(3)$ triplet, labeled (u, d, s) ("up," "down," "strange") in which the (u, d) pair formed an I -spin doublet and the s a singlet. Thus the mesons and baryons had the quark content

$$\begin{aligned} \pi^+ &\sim u\bar{d}, \quad \pi^- \sim d\bar{u}, \quad K^+ \sim u\bar{s}, \quad K^0 \sim d\bar{s}, \dots, \\ p &\sim uud, \quad n \sim udd, \quad \Lambda \sim uds, \dots, \\ \Delta^{++} &\sim uuu, \quad \Sigma^+ \sim uus, \dots, \quad \Omega^- \sim sss. \end{aligned} \quad (1.33)$$

$SU(3)$ invariance requires that the masses of the three quarks be degenerate. The observed pattern of $SU(3)$ symmetry breaking, which maintains I -spin conservation, can be obtained by keeping the u and d quarks degenerate while splitting off the s quark. The electric charge assignments for the quarks are peculiar:

$$Q_u = \frac{2}{3}, \quad Q_d = Q_s = -\frac{1}{3}, \quad (1.34)$$

as is the assignment of baryon number $B = \frac{1}{3}$ to the quarks, especially since vigorous searches failed to turn up evidence for fractionally charged particles. Still, the model had some immediate success: the $SU(3)$ multiplication rules immediately explain the hadronic multiplets, since

$$\begin{aligned} \mathbf{3} \otimes \bar{\mathbf{3}} &= \mathbf{8} \oplus \mathbf{1}, \\ \mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} &= (\mathbf{6} \oplus \bar{\mathbf{3}}) \otimes \mathbf{3} = \mathbf{10} + \mathbf{8} + \mathbf{8} + \mathbf{1}. \end{aligned} \quad (1.35)$$

The suppression of certain classes of strong decays (the Okubo-Zweig-Iizuka rules)³⁰ could be simply explained by forbidding quarkless intermediate states in decays expressed in terms of quark lines. Thus the suppression of the

decay of the ϕ meson (a 3S_1 $s\bar{s}$ state) into the final state $\rho^+ \pi^-$ compared to the phase space disfavored $K\bar{K}$ final state is easily understood (Fig. 6). Similarly chiral symmetry $SU(3)_L \times SU(3)_R$ is a good symmetry to the extent that vector currents like $\bar{u}\gamma^\mu d, \dots$, as well as axial currents like $\bar{u}\gamma^\mu \gamma_5 d, \dots$, are conserved. This requires that there be no quark mass term in the Lagrangian.

The weak interactions of the hadrons are simply described in terms of quarks. The lepton theory is transcribed with the doublet $(\nu_e, e)_L$ replaced by $(u, d)_L$ and u_R and d_R assigned singlet status under the weak $SU(2)$. The only difference is that the weak hypercharge assignments must be different for the $U(1)$ interaction. Since the electric charge is related to the weak I -spin and weak hypercharge by

$$Q = I_3 + \frac{1}{2}Y, \quad (1.36)$$

the hypercharge is $\frac{1}{3}$ for the doublet, $\frac{4}{3}$ for u_R , and $-\frac{2}{3}$ for d_R . The lepton-quark symmetry is very suggestive, and the existence of the muon doublet suggested to S. L. Glashow and J. D. Bjorken³¹ in 1964 (before the discovery of the τ lepton) the existence of a fourth quark, the *c quark*, to serve as a charge $\frac{2}{3}$ partner to the charge $-\frac{1}{3}$ s quark. Glashow, with J. Iliopoulos and L. Maiani,³² examined the structure of neutral currents that would emerge out of considering the charged currents within a framework of a symmetry, and found that they predicted large strangeness-changing neutral currents, in complete disagreement with rather stringent experimental limits. They found this to be compelling evidence for the existence of the c quark, and the search for the associated quantum number "charm" began to be taken seriously. The discovery of the narrow J/ψ and ψ' resonances³³ (soon interpreted as $c\bar{c}$ bound states), aside from converting almost everybody to a belief in the reality of quarks, led to a successful search for hadrons containing a single charmed quark. The physics of heavy-quark bound states has been a very important part of experimental research in the period since 1974, and that spectroscopy is discussed in some detail in Sec. IV.

The c quark is significantly more massive than the u, d, s quarks, so that there is no expectation of evidence for an $SU(4)$ symmetry. Furthermore, by an extension of the Bjorken-Glashow quark-lepton symmetry just mentioned, the lepton pair (ν_τ, τ) would call for another pair of quarks (t, b) ("top," "bottom"), and it is very gratifying that the $b\bar{b}$ bound state [$\Upsilon(9460)$] and associated states have been discovered.³⁴ The spectroscopy of the $b\bar{b}$ states is also discussed in detail in Sec. IV, as are the expected properties of the $t\bar{t}$ system. It is worth mentioning that it is by no means obvious that it is the actual quarks d, s, b that are the weak isospin partners of u, c, t , if by these letters we denote the mass eigenstates. Since the weak interactions do not conserve strangeness and "beauty" (the quantum number associated with the b quark), they cannot really distinguish between d, s , and b , so that it is linear combinations of these that appear in the weak interactions. The correspondence between leptons and quarks is thus more generally written as

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \leftrightarrow \begin{pmatrix} u \\ d' \end{pmatrix}, \begin{pmatrix} c \\ s' \end{pmatrix}, \begin{pmatrix} t \\ b' \end{pmatrix}. \quad (1.37)$$

This mixing of the mass eigenstates was already apparent in the Cabibbo theory that only involved the d and s quarks. Here

$$d' = d \cos\theta_c + s \sin\theta_c,$$

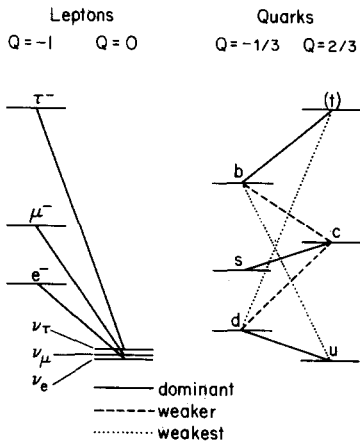


Fig. 7. Pattern of weak charge-changing transitions between lepton pairs and among quarks.

$$s' = -d \sin \theta_c + s \cos \theta_c \quad (1.38)$$

with $\theta_c \cong 13^\circ$ determined experimentally. With a six-quark scheme, the more general structure was written down by M. Kobayashi and T. Maskawa.³⁵ It is

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} c_1 & s_1 c_2 & s_1 s_2 \\ -s_1 c_3 & c_1 c_2 c_3 - s_2 s_3 e^{-i\delta} & c_1 s_2 c_3 + c_2 s_3 e^{-i\delta} \\ -s_1 s_3 & c_1 c_2 s_3 + s_2 c_3 e^{-i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{-i\delta} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix};$$

$$c_i = \cos \theta_i \quad i = 1, 2, 3. \quad (1.39)$$

$$s_i = \sin \theta_i$$

What is notable about this (most general) form is the appearance of the complex phase factor $e^{-i\delta}$ in it. Its presence signals a violation of invariance under time reversal, or equivalently (by the *CPT* theorem³⁶) a violation of *CP* invariance. Thus the observed effect,⁸ instead of being ascribed to an undetermined superweak interaction, finds its natural place within the theory of the weak interactions. The angles are small, but not well known, since they are measured in the suppressed decays (Fig. 7) of particles that are only now being discovered.

Renormalizability makes the lepton-quark symmetry quite compelling. In the weak interactions diagrams involving *VVA* (and *AAA*) vector and axial current vertices lead to divergences when they appear inside more complicated diagrams, and they cannot be renormalized. These divergences must be made to vanish, and the only way in which this can happen is through the cancellation between leptons and quarks circulating through the triangular loops³⁷ (Fig. 8). This cancellation is possible if for each lepton there exists a quark (masses play no role here) and if each quark is counted three times. This factor of 3 will be discussed below. In any case the requirement of renormalizability provides the first link between leptons and quarks that is expected to be found in the unified theory of the future.

The quark model had one difficulty: a three-quark wave function is expected to be totally antisymmetric by the Pauli principle. Yet a state like the Δ^{++} (the spin- $\frac{3}{2}$ I-spin- $\frac{3}{2}$ pion-nucleon resonance mentioned earlier) is symmetric in the quark and spin variables (for $J_z = \frac{3}{2}$ it is u, u, u_1) and for the lowest state in the **10**, the quarks are expected to be in relative *S* states. To avoid violation of the spin-statistics connection it was suggested that in addition to the *flavor* labels u, d, s, c, \dots , quarks should carry an additional three-value label called *color*.³⁸ Thus the wave function can be made totally antisymmetric by writing it in the form

$$\epsilon_{\alpha\beta\gamma} u_i^\alpha u_j^\beta u_k^\gamma, \quad (1.40)$$

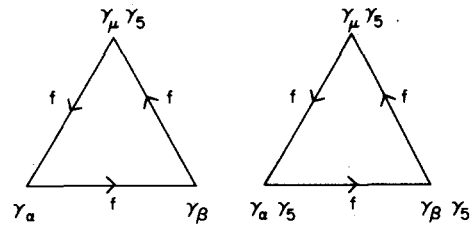


Fig. 8. Triangle graphs for vector-vector-axial and axial-axial-axial vertices, proceeding through intermediate fermion pair states.

where $\epsilon_{\alpha\beta\gamma}$ is totally antisymmetric in the indices and $\epsilon_{123} = 1$, and the summation is over the color variables. This is a singlet under the *SU*(3) group associated with the color variable. The absence of any evidence for color excitations (e.g., color octets) suggested the principle that physically observable states should be *SU*(3)_c singlets. Thus all baryonic states should be of the form

$$B_{ijk} \sim \epsilon_{\alpha\beta\gamma} q_i^\alpha q_j^\beta q_k^\gamma, \quad (1.41)$$

and all mesonic states, also color singlets, of the form

$$M_{ij} \sim \delta_{\alpha\beta} \bar{q}_i^\alpha q_j^\beta, \quad (1.42)$$

where i, j, \dots refer to the flavor. The evidence for three colors is impressive. We have already indicated that the required cancellation of the triangle anomalies required each quark to be counted thrice—once for each color. Another piece of evidence is also related to the triangular graph: one of the consequences of this graph is that a precise prediction of the lifetime of the π^0 can be made.³⁹ The rate is proportional to n_c^2 , where n_c is the number of colors. Agreement with experiment is obtained for $n_c = 3$. In addition, the currently accepted theory of the strong interactions (QCD) predicts that the reaction

$$e^+ + e^- \rightarrow \text{hadrons}$$

proceeds through the steps

$$e^+ + e^- \rightarrow \gamma \rightarrow q + \bar{q} \rightarrow \text{hadrons}$$

(Fig. 9). A consequence of this is that at high energies, when masses can be neglected, the relation

$$R \equiv \frac{\sigma(e^+ + e^- \rightarrow \text{hadrons})}{\sigma(e^+ + e^- \rightarrow \mu^+ + \mu^-)} = n_c \sum_{\text{quarks}} Q_c^2 \quad (1.43)$$

holds. The ratio, at energies when only u, d , and s quarks can be produced, is expected to be $n_c (\frac{4}{9} + \frac{1}{9} + \frac{1}{9})$, which agrees with the experimental value ≈ 2 when $n_c = 3$. A further test is the prediction

$$\frac{\Gamma(\tau \rightarrow \text{hadrons})}{\Gamma(\tau + \text{anything})} \cong \frac{2}{3}, \quad (1.44)$$

since the channels are

$$\tau^- \rightarrow e^- + \bar{\nu}_e + \nu_\tau$$

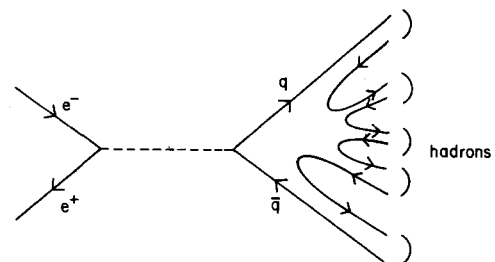


Fig. 9. Basic mechanism for the reaction $e^+ e^- \rightarrow \text{hadrons}$.

$$\rightarrow \mu^- + \bar{\nu}_\mu + \nu_\tau$$

$$\rightarrow d + \bar{u} + \nu_\tau \quad (3 \text{ colors of quarks}).$$

Experimentally the ratio is about 0.64.¹ The color symmetry, together with the proposal that only singlets are physical, lies at the core of the theory of strong interactions that will be discussed in Sec. II.

II. QUANTUM CHROMODYNAMICS AND ITS IMPLICATIONS

The discovery of the $SU(3)$ symmetry associated with the color degree of freedom, combined with the appeal of local gauge theories in which the interaction is uniquely fixed, inevitably led to the proposal that the $SU(3)_{\text{color}}$ symmetry be local.⁴⁰ Such a theory contains $3^2 - 1 = 8$ vector bosons, called *gluons*. The interaction is flavor independent, that is, u, d, s, c, b, t, \dots quarks have the same strong interactions. The gluons must be massless. Since there is no evidence for a long-range interaction associated with color, one of two things must happen. Either a kind of spontaneous symmetry breaking mechanism operates, in which the gluons become massive (which is what happens in the electroweak interactions), or the symmetry remains unbroken, but states that are not color singlets are unobservable except perhaps at very high temperatures and pressures, when a phase transition to a regime of free quarks and gluons becomes possible. Current prejudices favor the second option, known as *color confinement*, which is believed to be a consequence of the $SU(3)_{\text{color}}$ gauge theory. Support for this comes from models in which space-time is replaced by a lattice of points on which an analog of a gauge theory can be defined. Such models do lead to confinement. The problems associated with letting the lattice spacing approach zero, which is necessary to restore rotational and Lorentz symmetry, have not been solved. Since the details of the confinement mechanism are not yet understood, the theory cannot be tested in its predictions for hadron spectroscopy. It turns out that the coupling in quantum chromodynamics (QCD) is scale dependent, and becomes weak for large momentum transfers. Thus the theory can be tested at distances that are very short compared with the hadron size. This will be elaborated below.

The Lagrangian density for quarks interacting with gluons is

$$\mathcal{L} = \sum_{\text{flavors } f} \left[i \bar{q}_a \gamma^\mu \frac{\partial}{\partial x^\mu} q_a + g \bar{q}_a \gamma^\mu \left(\frac{\lambda_i}{2} \right)_{ab} A_\mu^i q_b - m_f \bar{q}_a q_a \right] - \frac{1}{4} \left(\frac{\partial A_\nu^i}{\partial x^\mu} - \frac{\partial A_\mu^i}{\partial x^\nu} + g f_{ijk} A_\mu^j A_\nu^k \right)^2 \quad (2.1)$$

$f = u, d, s, c, \dots$

The summation over repeated indices a, b, \dots referring to color is understood. The $\lambda_i/2$ are 3×3 matrix representations of the $SU(3)_c$ algebra generators, satisfying the commutation relations

$$[\lambda_i/2, \lambda_j/2] = i f_{ijk} \lambda_k/2. \quad (2.2)$$

The f_{ijk} are the $SU(3)$ structure constants. The flavor symmetry is only broken by the lack of degeneracy in the quark masses. There is a rather clean separation of the quarks into light and heavy quarks that is not understood. It manifests itself in two different kinds of spectroscopies, as can be seen by comparing the physics of the "ordinary" hadrons, made of the u, d, s quarks, and the hadrons that involve the heavy quarks c and b (and the as yet undiscovered t quark). For

the heavy-quark hadrons, the mass scale is set by the quark masses. For the light-quark hadrons, the situation is more complicated. There is evidence that the admixture of c and b quarks in the proton, say, that is primarily (uud), is of the order of a few percent, at most, and this suggests that for the discussion of the ordinary hadrons the heavy flavors can be omitted from (2.1). If the u, d, s, \dots quarks were massless, the Lagrangian in (2.1) would separate into two independent terms, one involving $q_{iL} \equiv \frac{1}{2}(1 - \gamma_5)q_i$, the other $q_{iR} \equiv \frac{1}{2}(1 + \gamma_5)q_i$ only. This implies an ungauged flavor $SU(3)$ symmetry for each term separately, and thus the Gell-Mann chiral symmetry²⁴ $SU(3)_L \times SU(3)_R$. The symmetry breaking has the form

$$\mathcal{L}_{SB} = m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s. \quad (2.3)$$

One may use this term to compute ratios of masses of the pseudoscalar mesons. For example,⁴¹

$$\frac{m_d}{m_u} = \frac{m^2(K^0) - m^2(K^+) + m^2(\pi^+)}{2m^2(\pi^0) + m^2(K^+) - m^2(K^0) - m^2(\pi^+)} \cong 1.8,$$

$$\frac{m_s}{m_d} = \frac{m^2(K^0) + m^2(K^+) - m^2(\pi^+)}{m^2(K^0) - m^2(K^+) + m^2(\pi^+)} \cong 20. \quad (2.4)$$

When this is combined with a rough estimate of m_s obtained from the $\Lambda - N$ mass difference, one gets numbers like

$$\begin{aligned} m_u &\cong 4.2 \text{ MeV}/c^2, \\ m_d &\cong 7.5 \text{ MeV}/c^2, \\ m_s &\cong 150 \text{ MeV}/c^2. \end{aligned} \quad (2.5)$$

These masses are significantly smaller than the "constituent" masses that are used in the quark phenomenology of hadrons discussed in Sec. III. Presumably the latter reflect the effects of the confinement mechanism. The masses in (2.5) are said to describe the "current" quarks, or equivalently, the bare quarks. They are very small on the scale of hadronic masses, and it may be a good approximation to set them equal to zero. The breaking of the chiral symmetry may arise from a dynamical Higgs mechanism that is purely chromodynamic in origin, or from spontaneously broken flavor dynamics, just as for leptons in the Weinberg-Salam model. In any case, it appears that the mass scale of the light quarks is much smaller than the natural scale for hadrons, and one may ask how one could ever get a result like $m_p = 1 \text{ GeV}/c^2$ out of a massless theory.

The parameter with the dimensions of a mass enters through renormalization. The theory is made finite by the introduction of a cutoff, the dependence on which can be isolated in the form of a renormalization constant that multiplies the amplitude under consideration. The separation into a constant that is divergent as the cutoff becomes infinite and a finite "renormalized" amplitude is arbitrary until a renormalization prescription is introduced. For a three-gluon vertex, for example, the finite factor can be fixed by the requirement that

$$\begin{aligned} \Gamma_{\mu\nu\rho}^{\text{finite}}(k_1, k_2, k_3) \Big|_{k_1^2 = k_2^2 = k_3^2 = -\mu^2} \\ = -g_s(\mu^2) [g_{\mu\nu}(k_1 - k_2)_\rho + g_{\nu\rho}(k_2 - k_3)_\mu \\ + g_{\rho\mu}(k_3 - k_1)_\nu], \end{aligned} \quad (2.6)$$

where the structure on the right-hand side is the one that emerges from the Lagrangian. The dependence on the prescription is stressed by writing $g_s = g_s(\mu^2)$. A change in that

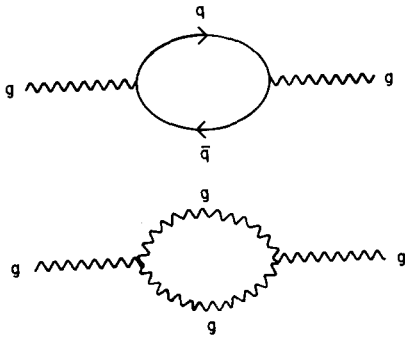


Fig. 10. One-loop contributions to the calculation of $\alpha_s(q)$. The gluon pair contribution contains both transverse and "Coulomb" gluons.

arbitrary parameter will lead to a change in g_s but not to a change in any physical quantity that is calculated in terms of $g_s(\mu^2)$. For a physical quantity $P(\mu, g_s(\mu^2))$ the equation⁴²

$$\mu \frac{d}{d\mu} P(\mu, g_s(\mu^2)) = \left[\mu \frac{\partial}{\partial \mu} + \left(\mu \frac{dg_s}{d\mu} \right) \frac{\partial}{\partial g_s} \right] P(\mu, g_s) = 0 \quad (2.7)$$

must hold. As a consequence of gauge invariance the same coupling constant dependence appears in the quark-gluon vertex, and for the coupling of a gluon with invariant square of the four-momentum q^2 , the dependence on q^2 just involves a calculation of vacuum polarization. With $\alpha_s(q^2) = g_s^2(q^2)/4\pi$ one finds that⁴³

$$\alpha_s(q^2) = \frac{\alpha_s}{1 + (\alpha_s/4\pi) [(11 - \frac{2}{3}n_f) \log(-q^2/\mu^2)]}. \quad (2.8)$$

Here α_s is the number chosen for $g_s^2/4\pi$ at $q^2 = -\mu^2$. The calculation is very similar to the calculation of vacuum polarization in quantum electrodynamics, and includes the diagrams shown in Fig. 10. The fermion pair contribution is proportional to the number of different flavors n_f that can be produced. The factor 11 arises from the three-gluon coupling. In quantum electrodynamics this factor is absent, and $\alpha(q^2)$ grows with $|q^2|$. The apparent singularity is not necessarily real, since with a growing coupling constant, the one-loop calculation is not trustworthy. In QCD, as long as $n_f < 16$, $\alpha_s(q^2)$ decreases for large $|q^2|$, and thus the one-loop calculation is valid. The theory therefore consistently predicts that the coupling becomes weak for large invariant momentum transfer squared. This is described as *asymptotic freedom* of the theory. It is convenient to rewrite (2.8) in the form

$$\alpha_s(q^2) = 12\pi / [(33 - 2n_f) \log(-q^2/\Lambda^2)] \quad (2.9)$$

with the new parameter Λ to be determined experimentally.

A consequence of asymptotic freedom is that under the right circumstances QCD is a calculable theory, and comparison with experiment is possible. Not all processes are amenable to this comparison. QCD is a theory of quark interactions, and the relationship of quark scattering to hadron scattering is indirect. Figure 11 shows that proton-proton elastic scattering involves vertices that depend on how quarks appear inside the proton, and these are not calculable in perturbation theory, since they involve momenta on the scale of a hadron mass, where the coupling is strong. For some processes one can show that the cross section factorizes into a short-distance part calculable in QCD,⁴⁴ and a long-distance part that is process independent and thus transferable from one class of processes to another. This has not been established for all processes.⁴⁵ In the remainder of this section we discuss some applica-

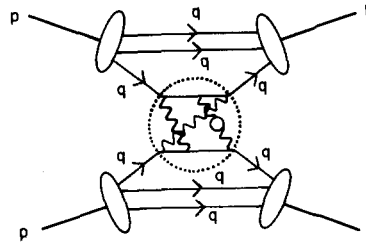


Fig. 11. Separation of proton-proton scattering into parts that involve the quark hadronic wave functions and the central quark-quark short-range scattering amplitude that is calculable in perturbative QCD.

tions and then conclude with a short description of the present status of the strong-coupling problem.

Electron-positron annihilation

The primary process is sketched in Fig. 12(a). The measurement of the total cross section includes all possible final states so that the particular mechanism of the conversion of quarks and gluons into hadrons (hadronization) is irrelevant here. The inclusion of gluon Bremsstrahlung [Fig. 12(b)] and gluon vertex and self-energy corrections to Fig. 12(a) modifies the result quoted in Eq. (1.43), so that⁴⁶

$$R(q^2) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 3 \sum_{\text{flavors}} Q_f^2 \left(1 + \frac{\alpha_s(q^2)}{\pi} + \dots \right). \quad (2.10)$$

More details about the reaction can be deduced if one assumes that the hadronization process does not involve large momentum transfers. In that case one expects the leading $q\bar{q}$ final state to lead to two narrow jets of hadrons emerging with an angular distribution characteristic of the production of a pair of spin- $\frac{1}{2}$ quarks,

$$\frac{d\sigma}{d(\cos\theta)} \propto (1 + \cos^2\theta), \quad (2.11)$$

where θ is the angle with respect to the initial beam direction. Furthermore, gluon bremsstrahlung is expected to give rise to a small admixture of three-jet events, with the gluon jet somewhat broader than the quark jets. The data from SLAC and DESY support these expectations.^{47,48}

Deep inelastic electron scattering

The reaction

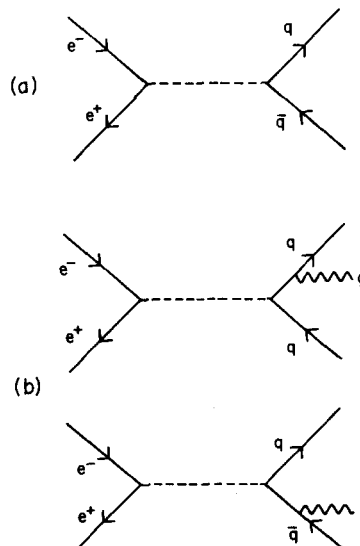


Fig. 12. Contributions to $\sigma(e^+e^- \rightarrow \text{hadrons})$: (a) quark pair (lowest order); (b) gluon bremsstrahlung.

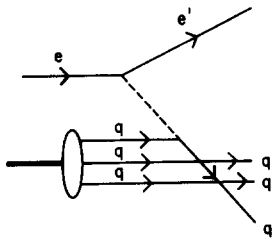


Fig. 13. Pictorial representation of parton model of deep inelastic electron scattering.

$$e + p \rightarrow e' + \text{hadrons}$$

in which an electron of energy E scatters off a proton through an angle θ , emerging with energy E' , is described by a cross section of the form⁴⁹

$$\frac{d^2\sigma}{d\nu dQ^2} = \frac{1}{m_p} \left(\frac{d\sigma}{dQ^2} \right)_{\text{point}} \left(2W_1(\nu, Q^2) \sin^2 \frac{\theta}{2} + W_2(\nu, Q^2) \cos^2 \frac{\theta}{2} \right). \quad (2.12)$$

Here $\nu = E - E'$ is the energy loss in the laboratory frame, and Q^2 is related to the scattering angle, because it is the invariant square of the electron momentum transfer, $Q^2 = -(p_e - p_{e'})^2$. The first factor is the cross section for elastic scattering of an electron off a point proton, and the two functions W_1 and W_2 contain dynamical information about the proton structure. Since the process is equivalent to

$$\gamma^* + p \rightarrow \text{hadrons},$$

where γ^* is a virtual photon, it is not surprising that there are two structure functions: they may be related to the absorption cross section for longitudinal and transverse virtual photons. For any fixed value of Q^2 the variable ν may be expressed in terms of the invariant square W^2 of the mass of hadrons in the final state, $W^2 = m_p^2 - Q^2 + 2m_p\nu$. In the region in which both ν and Q^2 are large, with

$$x = Q^2/2m_p\nu \quad (0 < x < 1) \quad (2.13)$$

held fixed, it was found that the two structure functions W_1 and νW_2 depend only on x to good approximation, provided x is not too close to 0 or 1. This *scaling law* had been predicted by J. D. Bjorken⁵⁰ and given a simple physical explanation by R. P. Feynman in his *parton model*.⁵¹ In that model, the proton, when viewed in a frame in which its momentum is very large, consists of a collection of independently moving entities, the *partons*. The inelastic scattering is described as the transfer of a large amount of four-momentum to a single parton (Fig. 13). The structure functions describe the probability of finding a parton that carries the fraction x of the total proton momentum. The ejection of the single parton destroys the coherence of the wave function that represents the proton, so that the final state consists of a large variety of hadronic states. It seems reasonable to assume that the partons are really the quarks. With this identification one finds that

$$\nu W_2(x) = 2m_p x W_1(x), \quad (2.14)$$

which is just a consequence of the spin $\frac{1}{2}$ of the partons (quarks). One also finds that the structure function may be written as

$$\begin{aligned} \nu W_2(x) &= x [Q_u^2(u(x) + \bar{u}(x)) + Q_d^2(d(x) + \bar{d}(x)) \\ &\quad + Q_s^2(s(x) + \bar{s}(x)) + \dots] \\ &= \frac{4}{3}xu(x) + \frac{1}{3}xd(x) + \dots, \end{aligned} \quad (2.15)$$

where $u(x), d(x), \dots$ are the probabilities of finding up-, down-, ... quarks with fractional momentum x inside the

proton. For a neutron the same functions appear with the interchange $u \leftrightarrow d$, since the proton is expected to be primarily the three-quark combination (uud) while the neutron is (ddu).

In the last line of (2.15) the "valence quark" terms have been isolated. The other contributions are called "sea quark" contributions, since they are not the primary components of the proton in the simple quark model and thus are vacuum fluctuation effects. The data show that the valence quarks dominate. For a proper analysis it is necessary to include data from the deep inelastic neutrino experiments,

$$\begin{aligned} \nu_\mu + p &\rightarrow \mu^- + \text{hadrons}, \\ \nu_\mu + n &\rightarrow \mu^- + \text{hadrons}, \\ \bar{\nu}_\mu + p &\rightarrow \mu^+ + \text{hadrons}, \\ \bar{\nu}_\mu + n &\rightarrow \mu^+ + \text{hadrons}. \end{aligned}$$

The qualitative shape of the probability functions is shown in Fig. 14. In the region $0.2 < x < 0.8$ the simple "valence" picture of the proton appears to hold, with the u quarks twice as important as the d quarks. The sea quarks only contribute in the region of small x . This is consistent with expectations from QCD, according to which a quark is expected to maintain most of its momentum when radiating the gluons that ultimately create the quark-antiquark pairs that populate the sea. The quark distribution functions may also be used to calculate the fraction of the momentum of the proton that is carried by the quarks (valence and sea)⁵²:

$$\langle x \rangle_{\text{quarks}} = \int_0^1 dx x(u(x) + \bar{u}(x) + d(x) + \dots). \quad (2.16)$$

This number turns out to be of the order of 0.5, indicating that about half of the proton momentum is carried by something that is flavor neutral, presumably the gluons.

One of the major achievements of QCD is that it justifies the parton model.⁴³ The detailed application of asymptotic freedom does involve a separation of long- and short-distance effects, but when done properly it yields definite predictions, including the predictions of *violations of scaling*⁵³ that must necessarily occur because of gluon effects (Fig. 15). The theory also works for the process

$$p + p \rightarrow \mu^+ + \mu^- + \text{hadrons}$$

in which the invariant mass of the lepton pair is large. Here the so-called Drell-Yan⁵⁴ mechanism operates, in which a quark from one of the protons annihilates a corresponding antiquark in the other proton, leading to a virtual photon that then decays into a lepton pair. Because the antiquark content of the proton is so small, the input functions

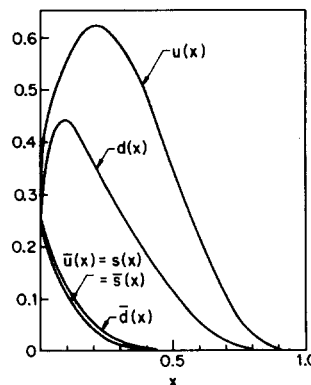


Fig. 14. Sketch of valence and sea quark distributions in proton [after G. C. Fox, Nucl. Phys. B **131**, 107 (1977), Table 3].

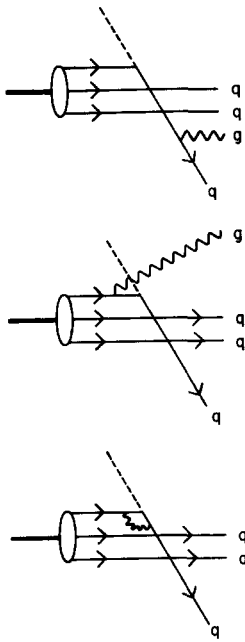


Fig. 15. Radiative corrections to deep inelastic electron scattering. The gluon bremsstrahlung contributions from the first two graphs lead to observable gluon jets when the gluons are highly energetic.

$\bar{u}(x), \bar{d}(x), \dots$ are imprecisely known from the deep inelastic scattering data, and thus comparisons with experiment are so far of only qualitative value. (This situation will improve when the process is studied in $p\bar{p}$ collisions.) QCD allows one to prove factorization, and the $\alpha_s(q^2)$ corrections have been computed. For technical reasons the factorization cannot be proved for elastic large-angle proton-proton scattering (see Fig. 11). To the extent that a parton picture is assumed to work, the data can be explained, when gluon contributions are taken into account.⁵⁵ In summary, the applications of perturbative QCD in the domain of its validity have not met with any setbacks, and in the case of inelastic lepton scattering QCD can be said to provide a quantitative fit to some aspects of the data.

Confinement effects

The situation regarding long-range (confinement) effects is less clear. There are qualitative ideas that have some physical consequences that are in agreement with what is observed. The qualitative ideas about the nature of confinement tend to agree that the quarks are bound by "strings" or tubes of color flux. The mechanism that gives rise to these strings is in dispute. One picture that has attracted a great deal of attention is the following⁵⁶: as a consequence of the strong coupling, the true vacuum state (the lowest energy state) is a somewhat complicated condensate of gluons and perhaps light quark-antiquark pairs. There is an analogy with the ground state of a superconductor. There the condensate of paired electrons gives rise to the Meissner effect, in which magnetic flux lines are excluded from the condensate, unless the energy balance favors a local breakdown to the normal phase. In such a "vacuum," a magnetic monopole (if such existed) and its antiparticle could not interact via the usual spread out dipole field. The energetically favored configuration would have a strongly localized "normal" region connecting the pair, in which the magnetic flux lines would be confined (Fig. 16). This would give rise to a potential energy proportional to the separation of the monopole and antimonopole, and thus to a confining potential. Color confinement could arise from a kind of "electric Meissner effect." The flux lines due to two

sources of color (say, a quark and an antiquark) are similarly localized inside a tube or *string*, and the mechanism is dual to the superconductivity mechanism: it is due to a condensate of magnetic monopoles. There is no need to assume to existence of real monopoles, since the classical equations of motion of QCD have among their solutions some that have all the properties of magnetic monopoles (electric and magnetic here refer to color fields). These ideas are still very much at a speculative stage, but the emerging string picture has some very attractive features. For example, in electron-positron annihilation, as the produced quark-antiquark pair separates, it becomes energetically favorable with increasing separation for the string to break and produce another quark-antiquark pair. This process may be imagined to go on until the typical spacing in the string approaches the typical hadronic size, ~ 1 fm ($\sim 5 \text{ GeV}^{-1}$). If the pair forms a hadron of typical energy ~ 1 GeV, then the original string would have had a linear energy density of order

$$k \simeq \frac{1 \text{ GeV}}{5 \text{ GeV}^{-1}} = 0.2 \text{ GeV}^2. \quad (2.17)$$

The linear Regge-Chew-Frautschi relation

$$J = \alpha_0 + \alpha' M^2, \quad \alpha' \simeq 0.8 - 0.9 \text{ GeV}^{-2} \quad (2.18)$$

mentioned earlier in this article provides some evidence for the string picture.⁵⁷ Imagine two massless quarks connected by a string of length r_0 with energy density k per unit length. For states with highest angular momentum (we neglect the spin of the quark, or deal with singlet states), the ends of the string are taken to rotate with the speed of light. Thus each point on the string has local speed

$$v/c = 2r/r_0. \quad (2.19)$$

(See Fig. 17.) The total mass is then

$$M = 2 \int_0^{r_0/2} \frac{k dr}{(1 - v^2/c^2)^{1/2}} = \frac{kr_0\pi}{2}, \quad (2.20)$$

and the orbital angular momentum is

$$L = 2 \int_0^{r_0/2} \frac{krv dr}{(1 - v^2/c^2)^{1/2}} = \frac{kr_0^2\pi}{8} = J. \quad (2.21)$$

Hence $J = M^2/2k\pi$ and

$$k = 1/2\pi\alpha'. \quad (2.22)$$

With the measured α' this yields $k = 0.18 - 0.20 \text{ GeV}^2$, in agreement with our earlier crude estimate.

The low angular momentum hadrons are less stringlike, and there a better qualitative picture is one in which color flux and light quarks form a bubble inside the vacuum. Such a picture is quantitatively described by the MIT bag model.⁵⁸ A region of space with volume Ar containing chromoelectric flux is assumed to contribute

$$E_{\text{bag}} = BV = BA r \quad (2.23)$$

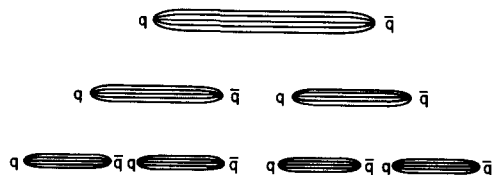


Fig. 16. Electric color lines connecting quark and antiquark in the QCD vacuum. When the sources are separated the breaking of the string accompanied by further quark-antiquark pair production takes place.

to the total energy. The constant B is taken to be a universal constant in the theory, and this term acts to compress flux lines into as small a volume as possible. The energy density of the chromoelectric field is $\frac{1}{2}\mathcal{E}^2$ and if the source of \mathcal{E} is a color charge Q , then by Gauss's law,

$$|\mathcal{E}| = g_s Q / A. \quad (2.24)$$

Thus the field energy is

$$E_{\text{field}} = \frac{1}{2}\mathcal{E}^2 A r = 2\pi\alpha_s Q^2 r / A. \quad (2.25)$$

The total energy $E_{\text{bag}} + E_{\text{field}}$ for a given r can be minimized by an appropriate choice of A , which corresponds to $E_{\text{bag}} = E_{\text{field}}$ and

$$A_{\text{min}} \equiv A_0 = \left(\frac{2\pi\alpha_s Q^2}{B} \right)^{1/2} \equiv \pi R_0^2, \quad (2.26)$$

where R_0 is the radius of the "bubble" of length r . The energy per unit length in this picture is

$$k = \frac{E_{\text{total}}}{r} = 2BA_0 = \frac{4\pi\alpha_s Q^2}{A_0} = \frac{16\alpha_s}{3R_0^2} \quad (2.27)$$

when $Q^2 = 4/3$ for the $SU(3)$ singlet quark-antiquark pair is used. With $\alpha_s \sim 1$, R_0 is of the order of a fermi. A flux tube of length r much larger than R_0 will break up into quark-antiquark pairs. If the length is chosen equal to R_0 , then the typical hadronic mass is

$$M \cong k\pi R_0 / 2 \quad (2.28)$$

according to (2.20). The qualitative point that emerges is that the choice of any one of the unknown parameters leads to reasonable values for all the other parameters, and that there is compatibility of the string picture with the bag limit.

A number of more direct assaults on the underlying QCD theory are under active study. We list a few of them.

1/N expansion⁵⁹

QCD can be defined for an arbitrary number of colors N . Since the theory is scale invariant, there are, as noted before, no naturally small parameters in the theory (the coupling at high energies is small, but how high the energy has to be must be determined from experiment), and it has been suggested to create a small parameter by working with N large, and $g_s = g/N^{1/2}$, with g fixed. The model has some attractive features: the combinatorics of summing over closed loops with the indices now going from 1 to N (or $N^2 - 1$) tend to favor certain routings of color flow in Feynman graphs. In particular, leading contributions come from planar Feynman graphs, in which only one quark loop, forming the boundary of the graph, can appear. If one assumes confinement, one finds that the quark-antiquark system has an infinite number of color singlet bound states that are stable to order $1/N$. This bears some resemblance to the meson spectrum. The picture of baryons consisting of N quarks bears less resemblance to the real world, and little progress has been made in summing up the planar graphs.

Semiclassical approximations⁶⁰

There exist formal expressions for scattering amplitudes that are nonperturbative and that involve some mathematically ill-defined expressions in which all possible classical field configurations are summed over. A typical expression

will involve

$$\sum_{\text{field configurations}} \exp\left(-\int d^4x \mathcal{L}(A_\mu^a(x), \bar{q}(x), q(x), \dots)\right), \quad (2.29)$$

after an analytic continuation to imaginary times (Euclidean rather than Minkowski geometry) has been made. Such expressions appear in statistical mechanics, and a new source of physical intuition is thus made available. The approximation method suggested by this formulation is an expansion about what is guessed to be a dominant set of field configurations. Configurations that represent a certain class of solutions of the classical Yang-Mills equations have been used. The nonperturbative effects associated with such solutions (the instantons) have had some qualitative successes in providing a model for the spontaneous breaking of chiral symmetry and in explaining the nature of the transition between the weak coupling and the strong coupling regimes, but they shed no light on confinement.

Lattice gauge theories⁶¹

It is possible to formulate QCD and other gauge theories on a cubical lattice with the gauge fields appearing quite directly as gauge-invariant dynamical variables associated with the links between adjacent lattice points. Because of gauge invariance link variables must either form closed loops or begin and end on color sources, e.g., quarks. In the strong coupling limit the energy is minimized by the smallest number of links, and for a quark-antiquark system the picture involves just a single string tying them together. Since there is a fixed amount of energy associated with each link, one gets the usual string picture with the linear confinement potential in the strong coupling limit. There is a fundamental difficulty: in order to approach the continuum limit (and thus restore Lorentz invariance, for example) the coupling has to be decreased as required by renormalization arguments given earlier, and then more and more closed loops of color flux can appear without increasing the energy. The problem of taking these into account becomes intractable. Recently such calculations have been performed by Monte Carlo methods⁶² in which loop configurations are sampled rather than summed over. These calculations are very encouraging, in that they show that there is a continuous transition between the strong and weak coupling regimes. The relation between the string tension, defined in the strong coupling "string" domain, and the size parameter of perturbation theory (2.9) is specified for $SU(2)$ and $SU(3)$ quarkless gauge theories⁶³:

$$A \cong 0.4\sqrt{k}, \quad (2.30)$$

and with $k \cong 0.2$ this leads to a number that is in qualitative agreement with what is obtained from fits to deep inelastic lepton scattering. It should be noted that these calculations do not include the effect of quarks. The inclusion of fermions in lattice gauge theories is beset with ambiguities, and remains an important unsolved problem.

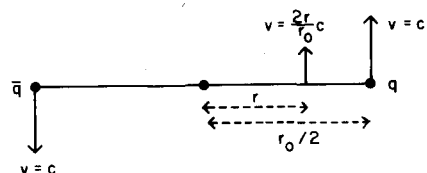


Fig. 17. Picture of rotating string with massless quarks.

Table III. Quark-antiquark composites with $L < 2$.

L	S	J^{PC}	Notation	Possible $^{2S+1}L_J$ admixtures
0	0	0^{-+}	1S_0	
0	1	1^{--}	3S_1	3D_1
1	0	1^{+-}	1P_1	
1	1	0^{++}	3P_0	
1	1	1^{++}	3P_1	
1	1	2^{++}	3P_2	3F_2
2	0	2^{-+}	1D_2	
2	1	1^{--}	3D_1	3S_1
2	1	2^{--}	3D_2	
2	1	3^{--}	3D_3	3G_3

In summary, there is every indication that the local gauge theory of color, QCD, is the correct theory of the strong interactions, and this encourages a serious phenomenological approach that may not immediately rest on QCD but that maintains constant contact with it. This phenomenology is discussed in Secs. III and IV.

III. LIGHT-QUARK SPECTROSCOPY

Perturbative quantum chromodynamics shows that high-energy collisions can be used to study the "bare" quarks. Until there is deeper understanding of strong coupling physics, information on how quarks make up hadrons must be obtained by looking at models of hadrons that are more tractable than the QCD theory. Even in this phenomenological domain there is a difficulty in treating hadrons made of light quarks, since relativistic effects are important. Fortunately, it appears that these conspire within the confinement mechanism (whatever that may turn out to be) to convert the "bare" quarks into *constituent quarks* whose masses are such that the use of nonrelativistic methods becomes more plausible. Within this framework, many properties of hadrons can be understood in very simple terms. The nonrelativistic limit is approached much more closely in the systems that involve heavy quarks, and there is hope that the bridging of the gap between phenomenology and fundamental theory will arise in the field of very-heavy-quark spectroscopy, which is the subject matter of Sec. IV.

Composite quark systems

The rules for identifying the quantum numbers of systems composed of quarks and antiquarks are quite straightforward. The way in which these composites are identified are as follows: we couple the quarks' spins together to give a certain total spin S , and then couple this total spin to the orbital angular momentum L to obtain the total angular momentum J . It is not assumed that L and S are separately conserved, so that there are very occasional ambiguities. These occur less frequently than one might think.

Consider quark-antiquark systems. The relative parity of a particle and its antiparticle is known to be negative, so that the parity of a $q\bar{q}$ system with orbital angular momentum is

$$P(q\bar{q}, L, S) = (-1)^{L+1}. \quad (3.1)$$

A neutral $q\bar{q}$ system is an eigenstate of the charge conjugation operator C , with eigenvalues⁶³

$$C(q\bar{q}, L, S) = (-1)^{L+S}. \quad (3.2)$$

Even though C is only defined for neutral systems we shall label all composites $q_i\bar{q}_j$ by J^{PC} . The composites formed in this way are listed in Table III and in Fig. 18. The ambiguities referred to above are severely restricted by P and C . P forbids mixing of even and odd values of L , and C then requires S (for $q\bar{q}$ states) to be unique. The cases in which mixing of states is allowed are shown in the table. The series $J^P = 0^+, 1^-, 2^+, 3^-, \dots$ is known as *natural parity* (it is the series that could be formed out of two spinless mesons), while the series $0^-, 1^+, 2^-, 3^+, \dots$ is called *unnatural*.

Some combinations of J^{PC} cannot be made out of $q\bar{q}$. The state 0^{--} is forbidden, as is the whole sequence $0^{+-}, 1^{-+}, 2^{+-}, \dots$. The first is forbidden since it requires $S=L$ and hence $C=+$. The sequence has $CP = -1$, and hence $S=0$, which implies $J=L$, that is $P = (-1)^{J+1}$. But the sequence has $P = (-1)^J$. These "C-exotic" states are only forbidden as $q\bar{q}$ composites, and if they exist they provide evidence for more complicated configurations such as $q\bar{q} + \text{gluon}$, $2q2\bar{q}$, or gluonic bound states.

The non-neutral $q\bar{q}$ composites fall into two classes with respect to mixing. Nonstrange composites with $Q \neq 0$ ($u\bar{d}, d\bar{u}$) are related to neutral ones ($u\bar{u} - d\bar{d}$)/ $\sqrt{2}$ by I spin, which is a very good symmetry. Thus the selection rules appropriate to the neutral member also apply to the charged ones. All members of an I -spin multiplet have the same G parity, which is most easily written for the neutral member as $G = C(-1)^I$, where I is the isospin. Since $G = -1$ for the pion, the G parity counts the number of pions into which the state decays. It is worth mentioning that if a state decays to both an even and an odd number of pions, then at least one class of decays violates isospin and must be caused by the electromagnetic interaction.

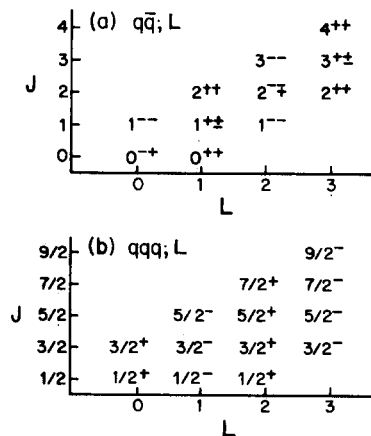


Fig. 18. Spins and parities of states composed of (a) $q\bar{q}$ (J^{PC} shown) and (b) qqq (J^P shown).

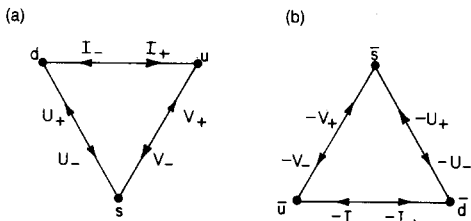


Fig. 19. I -spin, U -spin, and V -spin operators that change one quark flavor to another: (a) quarks; (b) antiquarks. The signs in (b) are an extension to $SU(3)$ of the convention that the antiparticle isodoublet corresponding to (u, d) is $(\bar{d}, -\bar{u})$.

For baryons all quarks have positive parity so that the parity is that of the spatial wave function, while the possible spins of three quark states are $S = \frac{1}{2}, \frac{3}{2}$. The composite states are shown in Fig. 18(b).

Elements of flavor $SU(3)$ and $SU(6)$.

We next discuss the internal quantum numbers of hadrons that can be made out of the "light" quarks u, d, s . These are taken to form a flavor triplet, or three-dimensional representation of $SU(3)$. The states may be transformed into one another by raising and lowering operators in $SU(3)$ that are analogous to the isospin raising and lowering operators, and are called I spin, U spin, and V spin. The quarks and the effects of these operators are shown in Fig. 19.⁶⁴ The mesons may be constructed by starting with a suitable state, such as $\pi^+ = u\bar{d}$ in Fig. 20, and "stepping" to all other members. It turns out that one must "step" in more than one direction to construct all the neutral members. Stepping down from π^+ yields π^0 , while stepping in the U_- direction from the K^0 yields a linear combination of π^0 and another state. The state that is orthogonal to π^0 is the (normalized) $I = 0$ state

$$\eta_8 = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}. \quad (3.3)$$

An octet of mesons has thus been constructed. There is a ninth $q\bar{q}$ state with $I = 0$,

$$\eta_1 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}, \quad (3.4)$$

which is orthogonal to every member of the octet. What has been accomplished is the decomposition

$$3 \otimes \bar{3} = 8 \oplus 1. \quad (3.5)$$

It may happen that neutral octet and singlet $I = 0$ mesons mix with one another so that the physical combinations are those with or without strange quarks. Thus one often sees combinations like

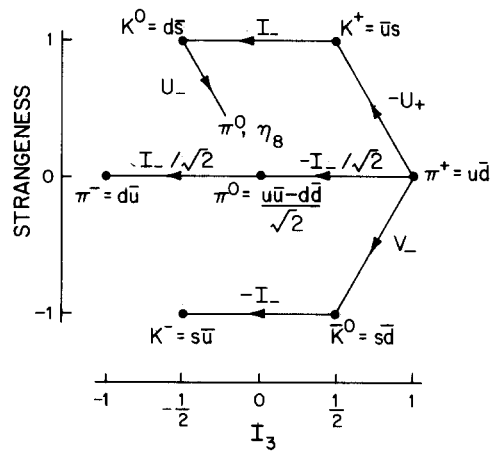


Fig. 20. Construction of the pseudoscalar meson octet with the help of stepping operators. The I -spin stepping operators take into account the Wigner coefficients appropriate to $I = 1$. The eighth state η_8 is defined in (3.3).

$$\begin{aligned} \omega &= (u\bar{u} + d\bar{d})/\sqrt{2} = \sqrt{(2/3)}\eta_1 + \sqrt{(1/3)}\eta_8, \\ \phi &= s\bar{s} = \sqrt{(1/3)}\eta_1 - \sqrt{(2/3)}\eta_8. \end{aligned} \quad (3.6)$$

When the spins of the quarks are put together as in Table III one obtains the mesons shown in Table IV. While some of the assignments in this table are not unique [$qq\bar{q}\bar{q}$ objects or gluonic bound state have been suggested for $\delta(980)$ and $E(1418)$, respectively, for example], most of the observed low-mass mesons are accounted for in the quark model. The remaining low-mass mesons listed in Ref. 1 but not mentioned here appear mainly to be radial excitations, such as the $\rho'(1600)$, which is believed to be a radial excitation of the $\rho(776)$. Some mesons of higher spin, candidates for ${}^3L_{L+1}$ assignments, are shown in Fig. 21.

A similar approach works for baryons. Let us examine the $SU(3)$ representations that can be constructed from three quarks. Starting from $\Delta^{+++} = uuu$ one may construct nine other states, all totally symmetric in the flavor indices. These are shown in Fig. 22.

While there is only one uuu state, there are two $I_3 = 1$ states having the same I_3 and strangeness as the totally symmetric Δ^+ but orthogonal to it. These are

$$P_{(a)} \equiv [ud]u = (udu - duu)/\sqrt{2}, \quad (3.7)$$

$$P_{(s)} \equiv (udu + duu - 2uud)/\sqrt{6}. \quad (3.8)$$

The subscripts (a) and (s) stand for antisymmetry or symmetry, respectively, in the first and second quarks, and the

Table IV. Mesons composed of light quarks, $L < 2$. (Numbers denote average masses in MeV.)^a

$2S + 1L_J$	J^{PC}	$I = 1$	$I = 1/2$	$I = 0$	$I = 0$
		$u\bar{d},$ $(u\bar{u} - d\bar{d})/\sqrt{2},$ $d\bar{u}$	$u\bar{s}, d\bar{s}$ $s\bar{u}, s\bar{d}$	$(u\bar{u} + d\bar{d})/\sqrt{2}$ or $^+\eta_8$	$s\bar{s}$ or $^+\eta_1$
1S_0	0^{-+}	$\pi(138)$	$K(496)$	$\eta(549)^+$	$\eta'(958)^+$
3S_1	1^{--}	$\rho(776)$	$K^*(892)$	$\omega(782)$	$\phi(1020)$
1P_1	1^{+-}	$B(1231)$	$Q_2(\sim 1400)$	$H(1190)$?
3P_0	0^{++}	$\delta(980)?$	$K(\sim 1400)?$	$\epsilon(\sim 1300)?$	$S^*(980)?$
3P_1	1^{++}	$A_1(1100)$	$Q_1(1280)$	$D(1284)^{+?}$	$E(1418)?$
3P_2	2^{++}	$A_2(1317)$	$K^*(1434)$	$f(1273)$	$f'(1516)$
1D_2	2^{-+}	$A_3(1640)$	$L(1765)$?	?
3D_3	3^{--}	$g(1688)$	$K^*(1784)$	$\omega(1668)$?

^a For more details, including references, see Ref. 75.

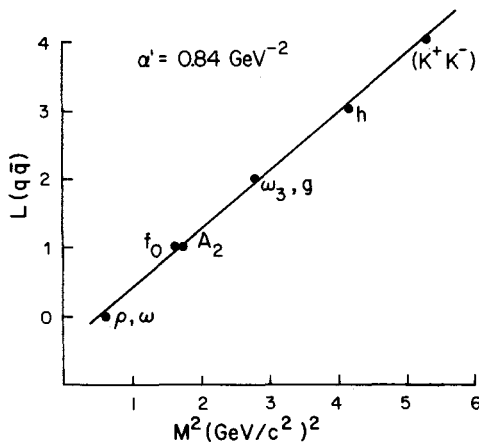


Fig. 21. Linear relation between orbital angular momentum L and squared mass M^2 for mesons composed of light (u, d) quarks.

symbol $[a_1 a_2 \dots a_n]$ stands for the totally antisymmetrized and normalized product of n objects. Both (3.7) and (3.8) generate octets of baryons when the stepping operations are applied. For example, (3.7), which turns out to be very convenient to use for $SU(3)$ calculations, gives rise to the states shown in Fig. 23. The eighth state is constructed in analogy with the η_8 discussed above:

$$A = ([us]d + [sd]u - 2[du]s)/\sqrt{6}. \quad (3.9)$$

Finally, a state can be constructed orthogonal to the $\mathbf{10}$ and the two $\mathbf{8}$'s: it is $A(1) = [uds]$ and is totally antisymmetric in flavor indices. The above construction corresponds to the group-theoretical decomposition of the product

$$3 \otimes 3 \otimes 3 = (6 \oplus \bar{3}) \otimes 3 = \mathbf{10} + \mathbf{8} + \mathbf{8} + \mathbf{1}. \quad (3.10)$$

Many of the baryons have two or three identical quarks, and if we wish to put $SU(3)$, spin, and spatial properties together while taking into account symmetry properties, it is best to use the group $SU(6)$,⁶⁵ which does the counting.

The two spin states of the three flavors of light quarks are taken to belong to the fundamental, six-dimensional representation of $SU(6)$. The baryon wave function $\Phi(123)$ must then be symmetric under the product of space and $SU(6)$ with respect to the interchange of any two quarks, because

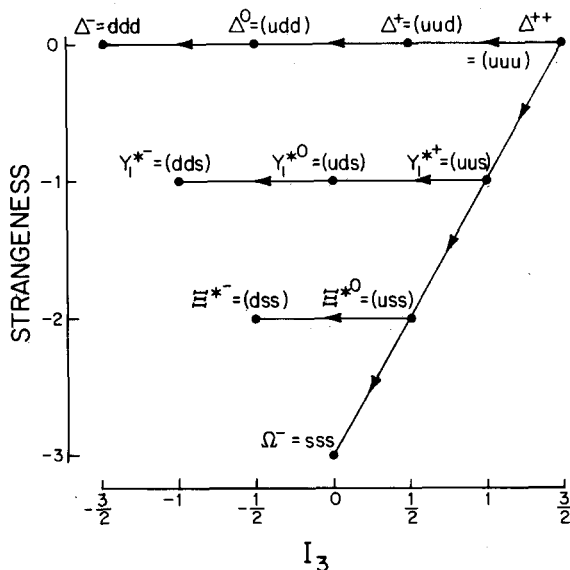


Fig. 22. Construction of the baryon decimet with the help of stepping operators. The parentheses denote symmetrized and normalized products, e.g., $(uds) = (uds + 5 \text{ permutations})/\sqrt{6}$.

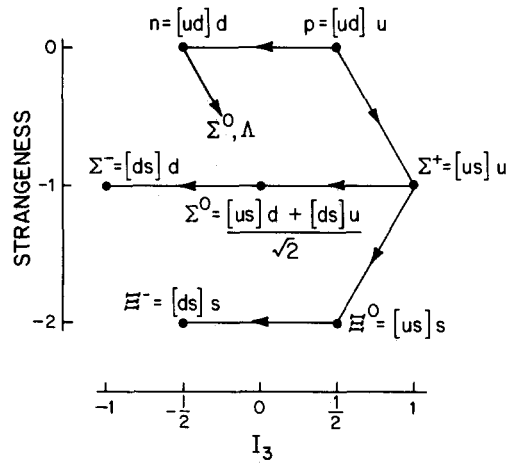


Fig. 23. Construction of the baryon octet with the help of stepping operators. Λ is defined in (3.9).

the color indices (here totally suppressed) take care of the required antisymmetry. Whereas for a two-particle system one need only worry about the symmetry properties under an exchange of the coordinates, for a three-particle system one must classify the behavior of the states under permutations acting on the coordinates. In the center-of-mass frame, in which the three coordinates r_1, r_2, r_3 obey

$$r_1 + r_2 + r_3 = 0, \quad (3.11)$$

a convenient choice of coordinates is the pair (λ, ρ) defined by

$$\begin{aligned} \lambda &= (r_1 + r_2 - 2r_3)/\sqrt{6}, \\ \rho &= (r_1 - r_2)/\sqrt{2}. \end{aligned} \quad (3.12)$$

A wave function that is symmetric in the three coordinates, for example, $\psi(r_1^2 + r_2^2 + r_3^2) = \psi(\lambda^2 + \rho^2)$, will not change sign under any of the permutations I (no permutation), (12), (13), (23), (123), and (132). An antisymmetric wave function, for example,

$$(r_1 \times r_2 + r_2 \times r_3 + r_3 \times r_1) \psi_{\text{symm}} \propto (\lambda \times \rho) \psi_{\text{symm}}$$

will change sign under (12), (23), and (13) but not under I , (123), and (132). There are, however, mixed representations, for which the transformations are more complicated, since, for example, under (12),

$$\begin{pmatrix} \lambda \\ \rho \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \lambda \\ \rho \end{pmatrix}, \quad (3.13)$$

and under (13)

$$\begin{pmatrix} \lambda \\ \rho \end{pmatrix} \rightarrow \begin{pmatrix} -1/\sqrt{2} & -\sqrt{3}/\sqrt{2} \\ -\sqrt{3}/\sqrt{2} & 1/2 \end{pmatrix} \begin{pmatrix} \lambda \\ \rho \end{pmatrix}. \quad (3.14)$$

Table V lists the transformations.

The construction of orbital angular momentum eigenfunctions can be carried out in a manner analogous to the two-particle construction. There one constructs homogeneous tensors in the relative coordinate r by starting (with $L_z = L$) with the function $(x + iy)^L$ corresponding to $L_z = L$ and successively applying L_- , the lowering operator, to it, to generate the eigenfunctions with $L_z < L$. For a three-body system the appropriate relative coordinates are

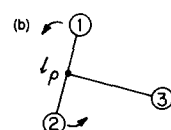
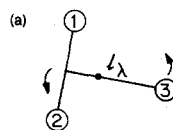


Fig. 24. Orbital excitations of the three-body system.

Table V. Permutation group action on three identical objects.

Representation	I	(12)	(13)	(23)	(123)	(132)
Symmetric S	1	1	1	1	1	1
Antisymmetric (A)	1	-1	-1	-1	1	1
Mixed ^a	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}$	$\frac{1}{2}\begin{pmatrix} -1 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$	$\frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$	$\frac{1}{2}\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}$

^a The matrices act on the basis $\begin{pmatrix} \lambda \\ \rho \end{pmatrix}$ where $\lambda = (r_1 + r_2 - 2r_3)/\sqrt{6}$ and $\rho = (r_1 - r_2)/\sqrt{2}$.

λ and ρ , and the angular momenta corresponding to these coordinates are depicted in Fig. 24.

The first few orbital excitations of three-body systems are listed in terms of λ and ρ in Table VI.⁶⁶ N denotes the degree of the tensor. In a harmonic oscillator model this would correspond to the excitation energy. For interquark forces that are not too different from a harmonic oscillator, N is still a rough index of excitation.

The spatial wave functions must be multiplied by $SU(6)$ wave functions of the appropriate symmetry. A tensor $T^{[ABC]}$, symmetric in all three indices, has 56 independent components: 6 ways of choosing all the same, $6 \cdot 5 = 30$ ways of choosing two indices the same, and $6 \cdot 5 \cdot 4 / (1 \cdot 2 \cdot 3) = 20$ ways of choosing them all different. Thus the spatially symmetric representation is 56 dimensional. A tensor $T^{[ABC]}$ antisymmetric in all three indices has all of them different and so is 20 dimensional. Finally, a tensor of mixed symmetry may be denoted by a pair $\{T^{[AB],C}, T^{[AB],C}\}$. Each component of the pair obeys a condition that ensures its orthogonality to $T^{[ABC]}$ and $T^{[ABC]}$, respectively, and can be shown to be 70 dimensional. Again, the decomposition is

$$6 \otimes 6 \otimes 6 = 56 \oplus 70 \oplus 70 \oplus 20. \quad (3.15)$$

The first few $SU(6)$ representations for $N \leq 3$ are summarized in Fig. 25. They correspond to the symmetry types enumerated in Table VI and Ref. 66.

The $SU(3) \times SU(2)$ content of the $SU(6)$ representations is easily worked out. The six-dimensional representation of $SU(6)$ is a product of an $SU(3)$ triplet (u, d, s) and $SU(2)$ doublet (spin up and down). We write this as $6 = (3, 2)$, with the two numbers denoting the $SU(3)$ and $SU(2)$ multiplicities, respectively. The product of three 6's is then, using (3.10),

$$6^3 = (10 \oplus 8 \oplus 8 \oplus 1, 4 \oplus 2 \oplus 2). \quad (3.16)$$

Table VI. Orbital excitations of three-body systems for $L < 2$.

N	L^P ^a	Symmetry	Tensor
0	0^+	S	1
1	1^-	M	(λ, ρ)
2	$\left\{ \begin{array}{l} 0^+ \\ 1^+ \\ 2^+ \end{array} \right.$	S	$\lambda^2 + \rho^2$
		M	$(\rho^2 - \lambda^2, 2\lambda \cdot \rho)$
		A	$\lambda \times \rho$
2	$\left\{ \begin{array}{l} 2^+ \\ \end{array} \right.$	S	$\lambda_i \lambda_j + \rho_i \rho_j - (1/3)\delta_{ij}(\rho^2 + \lambda^2)$
		M	$\rho_i \rho_j - \lambda_i \lambda_j - \text{trace}$
		M	$\rho_i \lambda_j + \rho_j \lambda_i - \text{trace}$

^aThe parity is that of the two orbital angular momenta $P = (-1)^{L_\rho + L_\lambda}$.

We have also used the fact that for spins $2 \otimes 2 \otimes 2 = (3 \oplus 1) \otimes 2 = 4 \oplus 2 \oplus 2$. To assign states to the representations listed in (3.15), it is easiest to first look at states with total symmetry in $SU(3)$ or $SU(2)$. Their symmetry in the remaining subgroup must be that of the $SU(6)$ representation to which they belong. Recall that for $SU(3)$ the symmetries are, according to (3.7)–(3.9),

$$10 = S, \quad 8 = M, \quad 1 = A; \quad (3.17)$$

while in $SU(2)$,

$$4 = S, \quad 2 = M. \quad (3.18)$$

Thus we learn that

$$\begin{aligned} (S, S) &= (10, 4) \text{ belongs to } 56(S), \\ (M, S) &= (8, 4) \text{ belongs to } 70(M), \\ (A, S) &= (1, 4) \text{ belongs to } 20(A), \\ (S, M) &= (10, 2) \text{ belongs to } 70(M). \end{aligned} \quad (3.19)$$

What are left are four (8, 2) and two (1, 2) pieces, and the only way to fit the pieces together is with

$$\begin{aligned} 56 &= (10, 4) + (8, 2), \\ 70 &= (10, 2) + (8, 4) + (8, 2) + (1, 2) \text{ (twice)}, \\ 20 &= (8, 2) + (1, 4). \end{aligned} \quad (3.20)$$

We can now identify some of the multiplets in Fig. 25 with specific states. This is done in Table VII. There are candidates for most of the N and Δ resonances, and gaps in other states are no cause for concern.^{67,68} The assignments to octets versus singlets are not unique for A 's, nor to octets versus decimets for Σ 's and Ξ 's, since mixing is possible, and in fact expected.⁶⁹ We shall give some elementary examples of this mixing in the next subsection.

Evidence for high- L states continues on to much higher masses than shown in Table VII. For example, the follow-

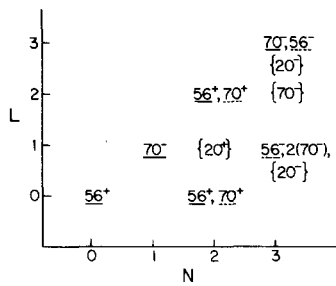


Fig. 25. $SU(6)$ multiplets as functions of L and degree N of tensor in λ and ρ . Superscripts denote parity. Underlines denote states for which there is firm evidence (solid line) or some evidence (dashed line). States in brackets cannot couple to ground-state baryons via single-quark transitions. For more details see Ref. 67.

ing identifications have been made:

$$\begin{aligned}
 N = 3 \quad 70, L = 3 & \begin{cases} N(2190) J^P = (7/2)^- \\ \Lambda(2100) J^P = (7/2)^- \end{cases} \\
 N = 4 \quad 56, L = 4 & \begin{cases} N(2220) J^P = (9/2)^+ \\ \Delta(2420) J^P = (11/2)^+ \end{cases} \\
 N = 5 \quad 70, L = 5 & N(2650) J^P = (11/2)^+ \end{aligned} \quad (3.21)$$

Some of these states can be put on Regge trajectories with lower mass states; the slopes are typically $\alpha' = 0.8 - 0.9 \text{ GeV}^{-2}$.

Masses

The strange quark is heavier than the u and d quarks. Strange hadrons will therefore differ in mass from non-strange ones because of (i) the quark mass difference itself; (ii) changes in the kinetic energy; (iii) changes in binding energies; and (iv) changes due to the mass dependence of other effects such as the hyperfine splittings. QCD has something to say about the pattern of hyperfine splitting. One can therefore discuss hadron masses in a way that goes beyond merely counting strange quarks.⁷⁰

We begin by just counting strange quarks. If that were all we would find that

$$*496 \text{ MeV} = K = (3\eta + \pi)/4 = 446 \text{ MeV}, \quad (3.22)$$

$$*776 \text{ MeV} = \rho = \omega = 783 \text{ MeV}, \quad (3.23)$$

$$*127 \text{ MeV} = \phi - K^* = K^* - \rho = 116 \text{ MeV}, \quad (3.24)$$

$$1192 \text{ MeV} = \Sigma = \Lambda = 1116 \text{ MeV} \quad (3.25)$$

and

$$(\Sigma \text{ or } \Lambda) - N = \Xi - (\Sigma \text{ or } \Lambda) \quad (254 \text{ or } 177 \text{ MeV}) = (125 \text{ or } 203 \text{ MeV}), \quad (3.26)$$

$$* \Sigma^* - \Delta = \Xi^* - \Sigma^* = \Omega^- - \Xi^* \quad (152 \text{ MeV}) = (150 \text{ MeV}) = (139 \text{ MeV}). \quad (3.27)$$

These relations are not bad as a first approximation. However, even in the early days of $SU(3)$ one could do better. Most of the relations (those denoted by an*) were first derived without reference to quarks, while (3.25) and (3.26) appeared in the combined form

$$*1129 \text{ MeV} = (N + \Xi)/2 = (3\Lambda + \Sigma)/4 = 1135 \text{ MeV}, \quad (3.28)$$

which is considerably better.

In group-theoretical language the starred relations follow from taking the mass operator to transform as the eighth component of an octet.⁷¹ This is equivalent to quark counting except for octet baryons, to which the mass operator can couple in two different ways. In Fig. 23 and (3.7) quarks in or out of "brackets" count differently. (This is equivalent to having both f - and d -type couplings as defined in Ref. 20.)

Corrections to the relations (3.25) and (3.26) that preserve (3.28) and all the other starred relations arise from hyperfine splitting. A simple calculation using single-gluon exchange shows that the Coulomb potential between a quark and an antiquark in a color singlet is

$$V_{q\bar{q}}(\mathbf{1}) = -\frac{4}{3}\alpha_s/r, \quad (3.29)$$

and between two quarks in a color $\bar{\mathbf{3}}$,

$$V_{q\bar{q}}(\bar{\mathbf{3}}) = -\frac{2}{3}\alpha_s/r. \quad (3.30)$$

In a baryon every quark pair is in a color $\bar{\mathbf{3}}$, since the pair must be coupled to a third quark to give a color singlet. A straightforward transcription of results for the hyperfine Coulomb interaction found in many textbooks⁷² yields the result that in a meson the hyperfine interaction between quarks and antiquarks is

$$\Delta E_{q\bar{q}}^{\text{HFS}}(\mathbf{1}) = \frac{8\pi\alpha_s}{9m_1m_2} |\psi(0)|^2 \sigma_1 \cdot \sigma_2, \quad (3.31)$$

Table VII. Baryons composed of light quarks, $L < 2$. Numbers denote average masses in MeV.

N	$SU(6), L$	$SU(3)$	J^P	States
0	56,0	8	$1/2^+$	$N(939), \Lambda(1116), \Sigma(1193), \Xi(1318)$
		10	$3/2^+$	$\Delta(1232), \Sigma(1385), \Xi(1533), \Omega(1672)$
1	70,1	1	$1/2^-$	$\Lambda(1405), \Lambda(1520)$
		8	$1/2^-$	$N(1535), \Lambda(1670), \Sigma(1750)$
			(two)	$N(1700), \Lambda(1870)$
			$3/2^-$	$N(1520), \Lambda(1690), \Sigma(1670), \Xi(1820)?$
			(two)	$N(1700), \Sigma(1940)?$
		10	$5/2^-$	$N(1670), \Sigma(1765)$
			$1/2^-$	$\Delta(1650), \Delta(1670)$
		2	56,2	8
8	$5/2^+$			$N(1688), \Lambda(1815), \Sigma(1915), \Xi(2030)?$
10	$1/2^+$			$\Delta(1910), \Delta(1890)$
	$3/2^+$			$\Delta(1890), \Delta(1950), \Sigma(2030)$
	$5/2^+$			$\Delta(1890), \Delta(1950), \Sigma(2030)$
56,0	8		$1/2^+$	$N(1470), \Sigma(1660)$
	10		$3/2^+$	$\Delta(1690)$

Table VIII. Meson masses with hyperfine splittings incorporated.

Meson	Coeff. of m_u or m_d	Coeff. of m_s	ΔE^{HFS}	Prediction (MeV/c ²)
$\pi(138)$	2	0	$-3a/m_u^2$	140
$K(496)$	1	1	$-3a/m_u m_s$	485
$\eta(549)$	2/3	4/3	$-a/m_u^2 - 2a/m_s^2$	559
$\rho(776)$ $\omega(783)$	2	0	a/m_u^2	780
$K^*(892)$	1	1	$a/m_u m_s$	896
$\phi(1020)$	0	2	a/m_s^2	1032

where m_1 and m_2 are the masses of the quarks, $|\Psi(0)|^2$ is the square of their relative wave function for zero separation, and σ_1, σ_2 are Pauli spin operators. The quark-quark potential (3.30) yields an expression equal to half of (3.31) for any pair of quarks in a baryon. We may thus make an extremely simple model for hadron masses:

$$M(\text{meson}) = m_1 + m_2 + a(\sigma_1 \cdot \sigma_2)/m_1 m_2, \quad (3.32)$$

$$M(\text{baryon}) = \sum_{i=1}^3 m_i + a' \sum_{i<j} (\sigma_i \cdot \sigma_j)/m_i m_j, \quad (3.33)$$

where the last term in (3.32) accounts in a phenomenological way for the hyperfine interaction. Here a and a' are treated as free parameters. To evaluate $\sigma_1 \cdot \sigma_2$ for a meson of total spin $S = S_1 + S_2 = \frac{1}{2}(\sigma_1 + \sigma_2)$ we use

$$S(S+1) = S^2 = \frac{1}{4}(\sigma_1 + \sigma_2)^2 = \frac{3}{2} + \frac{1}{2}\sigma_1 \cdot \sigma_2,$$

so that

$$\sigma_1 \cdot \sigma_2 = \begin{cases} -3 & \text{for } S=0 \\ +1 & \text{for } S=1 \end{cases} \quad (3.34)$$

The results of choosing $m_u = 310 \text{ MeV}/c^2$, $m_s = 483 \text{ MeV}/c^2$, and $a/m_u^2 = 160 \text{ MeV}/c^2$ are given in Table VIII. The large $\pi\rho$ splitting is ascribed to the hyperfine interaction; as can be seen, the model works remarkably well.

The baryon mass formula needs the evaluation of $\sigma_i \cdot \sigma_j$. If all three quarks have the same mass, one may use

$$\begin{aligned} \sum_{i<j} \sigma_i \cdot \sigma_j &= 4 \sum_{i<j} \mathbf{S}_i \cdot \mathbf{S}_j = 2[2S(S+1) - 3(\frac{3}{2})] \\ &= \begin{cases} +3 & \text{for } S = \frac{3}{2} \\ -3 & \text{for } S = \frac{1}{2} \end{cases} \end{aligned} \quad (3.35)$$

To take into account $m_s > m_u \simeq m_d$ in (3.33) we need to distinguish $\sigma_n \cdot \sigma_n$ ($n \equiv u, d$), $\sigma_n \cdot \sigma_s$, and $\sigma_s \cdot \sigma_s$. Let us give examples for Σ and Λ . In a Σ ($J^P = \frac{1}{2}^+$) the two nonstrange

quarks are in an S wave (symmetric) $I = 1$ (symmetric) color $\bar{\mathbf{3}}$ (antisymmetric) state, so that they must always be coupled up to spin $S = 1$. Thus $\sigma_n \cdot \sigma_n = 1$. By (3.35) $\sigma_n \cdot \sigma_n + 2\sigma_n \cdot \sigma_s = -3$ so that $\sigma_n \cdot \sigma_s = -2$. In a Λ ($J^P = \frac{1}{2}^+$) the nonstrange quarks are in an $I = 0$ (antisymmetric) hence $S = 0$ (antisymmetric) state: $\sigma_n \cdot \sigma_n = -3$ and $\sigma_n \cdot \sigma_s = 0$. Hence

$$\Delta E^{\text{HFS}} = \begin{cases} -3a'/m_u^2 & (\Lambda) \\ a'/m_u^2 - 4a'/m_u m_s & (\Sigma). \end{cases} \quad (3.36)$$

The Σ - Λ mass splitting is thus due to a difference in hyperfine effects. If we write similar expressions for N and Ξ and expand out (3.33) to first order in $m_s - m_u$, we recover the expression (3.28). A similar approach works for the baryon decimet ($J^P = \frac{3}{2}^+$). The masses implied by the above relations are summarized in Table IX.

The model is certainly satisfactory in view of all the effects that have been neglected: kinetic and binding energies, variations in $|\Psi(0)|^2$ and so on. What this means is that a large part of the mass pattern of the **56**-plet of baryons can be understood on the basis of quark mass differences and hyperfine interactions. The baryon parameters are $m_u = 363 \text{ MeV}/c^2$, $m_s = 538 \text{ MeV}/c^2$, and $a'/m_u^2 = 50 \text{ MeV}/c^2$. Although the effective masses of the quarks in baryons are not exactly the same as in mesons, their differences seem to be almost the same. The shifts in overall values may reflect small differences in binding effects.

Magnetic moments of baryons

We may use the success of Table IX to calculate the magnetic moments of the baryons on the assumption that these are just the vector sums of those of the quarks in them.

The two u quarks in a proton belonging to **56**, $L = 0$ are coupled to $S = 1$, by symmetry arguments made earlier.

Table IX. Baryon masses with hyperfine splittings incorporated.

Baryon	Coeff. of m_u or m_d	Coeff. of m_s	ΔE^{HFS}	Prediction (MeV/c ²)
$N(939)$	3	0	$-3a'/m_u^2$	939
$\Lambda(1116)$	2	1	$-3a'/m_u^2$	1114
$\Sigma(1193)$	2	1	$a'/m_u^2 - 4a'/m_u m_s$	1179
$\Xi(1318)$	1	2	$a'/m_s^2 - 4a'/m_u m_s$	1327
$\Delta(1232)$	3	0	$3a'/m_u^2$	1239
$\Sigma^*(1384)$	2	1	$a'/m_u^2 + 2a'/m_u m_s$	1381
$\Xi^*(1533)$	1	2	$a'/m_s^2 + 2a'/m_u m_s$	1529
$\Omega(1672)$	0	3	$3a'/m_s^2$	1682

Table X. Baryon magnetic moments predicted for static quarks. ^a

Baryon	Moment	Prediction (n.m.)	Experiment (n.m.)
p	$(4/3)\mu_u - (1/3)\mu_d$	2.79	2.793
n	$(4/3)\mu_d - (1/3)\mu_u$	-1.86	-1.913
Λ	μ_s	-0.58	-0.6138 ± 0.0047
$\Sigma^0 \rightarrow \Lambda$	$\frac{1}{\sqrt{3}}(\mu_d - \mu_u)$	-1.61	$-[1.82 \pm 0.25]$
Σ^+	$(4/3)\mu_u - (1/3)\mu_s$	2.68	2.33 ± 0.13
Σ^0	$(2/3)(\mu_u + \mu_d) - (1/3)\mu_s$	0.82	
Σ^-	$(4/3)\mu_d - (1/3)\mu_s$	-1.05	-1.41 ± 0.25
Ξ^0	$(4/3)\mu_s - (1/3)\mu_u$	-1.40	-1.253 ± 0.014
Ξ^-	$(4/3)\mu_s - (1/3)\mu_d$	-0.47	-0.69 ± 0.04

^a For detailed references to the experiments see Ref. 75, Table 14, and P. T. Cox *et al.*, Phys. Rev. Lett. 46, 877 (1981).

This $S = 1$ composite is coupled with the d quark to total spin $\frac{1}{2}$:

$$|p \uparrow\rangle = \sqrt{\frac{2}{3}}|uu, S_z = 1 \otimes d \downarrow\rangle - \sqrt{\frac{1}{3}}|uu, S_z = 0 \otimes d \uparrow\rangle. \quad (3.37)$$

Thus

$$\mu_p = \frac{2}{3}(2\mu_u) + (-\frac{2}{3} + \frac{1}{3})\mu_d = \frac{4}{3}\mu_u - \frac{1}{3}\mu_d. \quad (3.38)$$

A similar calculation applies to n , Σ^\pm , and the Ξ 's. The u and d quarks in a Λ belonging to $56, L = 0$ always couple to spin 0, as mentioned earlier, so that $\mu_\Lambda = \mu_s$. The u and d quarks in Σ^0 are coupled to spin 1 so that

$$\mu_{\Sigma^0} = \frac{2}{3}(\mu_u + \mu_d) - \frac{1}{3}\mu_s. \quad (3.39)$$

The $\Sigma^0 - \Lambda$ "transition moment" describing the rate for $\Sigma^0 \rightarrow \Lambda \gamma$ is

$$\mu_{\Sigma^0 \Lambda} = |e| \langle \Lambda \uparrow | \Sigma Q_i \sigma_{i3} / 2m_i | \Sigma^0 \uparrow \rangle. \quad (3.40)$$

The quark magnetic moments implied by the masses in Table IX are

$$\begin{aligned} \mu_u &= \frac{2}{3}|e|/2m_u = 1.863 \text{ nuclear magnetons,} \\ \mu_d &= -\frac{1}{2}\mu_u = -0.931 \text{ n.m.,} \\ \mu_s &= \mu_d(m_d/m_s) = -0.582 \text{ n.m.,} \end{aligned} \quad (3.41)$$

and the predictions for the baryonic magnetic moments are listed in Table X. The agreement of predicted and experimental magnetic moments is very good for p, n, Λ (within a few percent) and is within 0.2 nuclear magnetons for the Ξ 's. The discrepancy for Σ^+ is a bit worse, but also not serious in view of the crudeness of the model. Some progress has been made in understanding the departure of the observed Ξ moments from the predictions in Table X⁷³; the underlying physics appears to be very similar to the small

admixture of D wave in the dominant S -wave structure of the deuteron. It is worth pointing out that one does as well in Table X as in predicting the magnetic moments of nuclei from those of their constituent nucleons.

Meson radiative decays

A simple probe of the magnetic moments of quarks in a meson is provided by the rates for processes in which a photon is emitted via its coupling to the spin of the quark (magnetic dipole or $M1$ transitions). We omit details of these calculations that can be found in many places⁷⁴ and simply quote the results in Table XI. The calculations involve an adjustable overlap integral I between the initial and final wave functions. The table shows that for many processes $|I|^2 \sim \frac{1}{2}$. A universal value is certainly expected for processes in which the initial and final states have, respectively, similar masses. Such is the case for $\omega \rightarrow \pi^0 \gamma$ and $\rho \rightarrow \pi \gamma$. We expect

$$\frac{\Gamma(\omega \rightarrow \pi^0 \gamma)}{\Gamma(\rho \rightarrow \pi \gamma)} = \frac{(\mu_d - \mu_u)^2}{(\mu_d + \mu_u)^2} = 9, \quad (3.42)$$

raised to 9.4 by small kinematic corrections. The observed ratio is not much larger. The process $\pi + \gamma \rightarrow \rho$ has recently been observed at the Fermi National Accelerator Laboratory via the excitation of π in the Coulomb field of a nucleus (Primakoff effect). Even the processes involving strange quarks appear to be consistent with $|I|^2 \sim \frac{1}{2}$. The rate for $K^{*0} \rightarrow K^0 \gamma$ is from an old low-energy Primakoff effect measurement; a more precise measurement will be possible at Fermilab energies.

Many other results can be obtained for light-quark systems without a detailed knowledge of their dynamics. A

Table XI. Magnetic dipole ($M1$) transitions ($1^- \rightarrow 0^- + \gamma$) for mesons composed of light quarks. ^a

Process	Rate in units of $\omega^3 I ^2 / 3\pi$	Prediction (keV)	Experiment (keV)	$ I ^2$
$\omega \rightarrow \pi^0 \gamma$	$(\mu_u - \mu_d)^2$	$1390 I ^2$	890 ± 50	0.64 ± 0.04
$\rho \rightarrow \pi \gamma$	$(\mu_u + \mu_d)^2$	$148 I ^2$	67 ± 7	0.45 ± 0.05
$\omega \rightarrow \eta \gamma$	$(\mu_u + \mu_d)^2 / 2$	$11 I ^2$	3 ± 2.5	0.27 ± 0.23
$\rho \rightarrow \eta \gamma$	$(\mu_u - \mu_d)^2 / 2$	$92 I ^2$	50 ± 13	0.54 ± 0.14
$\eta' \rightarrow \omega \gamma$	$3(\mu_u + \mu_d)^2 / 2$	$17 I ^2$	7.6 ± 3	0.45 ± 0.18
$\eta' \rightarrow \rho \gamma$	$3(\mu_u - \mu_d)^2 / 2$	$171 I ^2$	83 ± 30	0.48 ± 0.18
$\phi \rightarrow \eta \gamma$	$2\mu_s^2$	$110 I ^2$	62 ± 9	0.56 ± 0.08
$\phi \rightarrow \pi^0 \gamma$	0	0	5.7 ± 2	
$K^{*+} \rightarrow K^+ \gamma$	$(\mu_u + \mu_s)^2$	$153 I ^2$	60 ± 15	0.39 ± 0.10
$K^{*0} \rightarrow K^0 \gamma$	$(\mu_d - \mu_s)^2$	$224 I ^2$	75 ± 35	0.34 ± 0.16

^a For detailed references see Ref. 75, Table 15.

very powerful tool for describing electromagnetic and strong decays of hadrons is the assumption that a single quark in the hadron changes its state, whether in photon or pion emission. Details can be found in Refs. 49 and 75.

Exotic combinations of light quarks

Color singlets can be formed of more quarks than just the familiar $q\bar{q}$ and qqq discussed so far. We call combinations such as $2q2\bar{q}$, $6q$, $(4q)\bar{q}$ *exotic*. Specific models, such as the MIT bag, permit the calculation of masses and decay widths of exotics.⁷⁶ One important effect involves the recoupling of quark spins to give mass eigenstates with respect to hyperfine interactions. The lowest $qq\bar{q}\bar{q}$ and $6q$ states thus can be more tightly bound than just a pair of mesons or baryons, if effects other than the hyperfine interaction do not raise the mass too much.⁷⁷ To illustrate the recoupling, we consider a simple example, a hypothesized bound state of two Λ hyperons.⁷⁸

Each Λ has a ud pair coupled up to spin zero, giving a hyperfine interaction energy

$$2E|_{\Lambda} = -6a'/m_u^2 \quad (3.43)$$

for two widely separated Λ 's. However, by recoupling quark spins one can do better: in the $SU(3)$ limit,

$$\Delta E|_{\Lambda\Lambda} = -9a''/m_q^2, \quad (3.44)$$

where $a'' \sim |\Psi(0)|^2$ and, presumably, since the six quarks are spread out over a larger volume, $a'' < a'$. To calibrate the amount of binding gained, we recall that $3a'/m_u^2 = 150$ MeV. Thus the two Λ 's could be bound by an energy of somewhat less than 150 MeV. If so, the $\Lambda\Lambda$ bound state would have to decay weakly. Explicit calculations yield a binding energy of 80 MeV. These calculations are based on the MIT Bag model, which takes into account the changes in kinetic energies and $|\Psi(0)|^2$ when two bags merge into a larger one. Preliminary data using $pp \rightarrow K^+K^+X$ shows no evidence for a state X below the $\Lambda\Lambda$ threshold.⁷⁹

Recoupling of quark spins also can lead to enhanced hyperfine interactions in $L=0$ $qq\bar{q}\bar{q}$ systems. It is interesting that the $qq\bar{q}\bar{q}$ exotics with the greatest hyperfine attraction have qq coupled to a flavor $\bar{3}$ and $\bar{q}\bar{q}$ coupled to a flavor 3 , so that from the standpoint of charges they look just like $q\bar{q}$ states. However, their couplings to decay products can be different. On this basis Jaffe has argued that $S^*(980)$ and $\delta(980)$ mentioned in Table IV should be $n\bar{n}s\bar{s}$ ($n = u$ or d) states, since they seem to couple so strongly to $K\bar{K}$.

Quarkless mesons

Gluons have a self-coupling and so should be able to form quarkless bound states. These will mix with $q\bar{q}$ or $q\bar{q} + \text{gluon}$ states, so it may be hard to identify them. Many suggestions have been made.⁸⁰ The decay

$$J/\psi \rightarrow \gamma + \text{anything} \quad (3.45)$$

should be a source of gluonic bound states (GBS) since it very likely proceeds via two gluons by the OZI (Okubo-Zweig-Iizuka³⁰) rules. The total rate for $J/\psi \rightarrow \gamma + (2 \text{ gluons})$ can be estimated to be a few percent. The spectrum for two free gluons should peak at low two-gluon effective masses but is in fact distorted towards higher effective masses. Perhaps this is an indication in favor of GBS around 2 GeV. In first approximation, lattice gauge theories suggest GBS masses of the order of four times typical

hadronic masses (Fig. 26) but more detailed calculations yield⁸¹

$$m_{\text{GBS}} = \begin{aligned} &(3.7 \pm 1.2)\sqrt{k} && SU(2) \text{ Monte Carlo} \\ &(2.4 \pm 1.2)\sqrt{k} && SU(2) \text{ 12-term strong coupling} \\ &(2.9 \pm 1.2)\sqrt{k} && SU(3) \text{ 12-term strong coupling,} \\ & && (3.46) \end{aligned}$$

which, with the accepted values of k , predicts 1.3 ± 0.5 GeV.

GBS of 1–2 GeV occur in bag models. Good candidates for mixing with ordinary hadrons are GBS with $J^{PC} = 2^{++}$ (mixing with f^0), and 0^{-+} [mixing with η , η' , $E(1420)$]⁸⁰. Clues might be differences in the resonance width, shape, and branching ratios of states produced under different circumstances, for example, for $J/\psi \rightarrow \gamma + f^0$ compared with $\pi^- p \rightarrow f^0 n$. Since GBS have to turn into hadrons by gluon-quark couplings, they should be narrower than ordinary hadrons but wider than quarkonia such as J/ψ and Υ , whose hadronic decays require more powers of these couplings. GBS with unusual J^{PC} can exist,⁸² but it may be hard to distinguish them from $qq\bar{q}\bar{q}$ or $q\bar{q} + \text{gluon}$ states without detailed dynamical calculations of the masses and widths of each.

IV. HEAVY QUARKS: CHARM, BEAUTY, AND BEYOND

The discovery of the J/ψ announced in November 1974⁸³ was a revolutionary event in elementary particle physics. The new, narrow state was immediately interpreted as the lowest bound state of a new quark and its anti-quark $c\bar{c}$. It strongly implied the existence of the c quark, which was desperately needed for the extension of the Weinberg-Salam theory to hadronic electroweak interactions, as first noted by Glashow, Iliopoulos, and Maiani (GIM).⁸⁴ It also opened a window on a new vista of high-energy physics, the domain of heavy quarks. Some aspects are still to be explored: that is the field of large masses and small momentum transfers, which may be accessible to perturbative QCD and provide some of the best tests of the theory. Others involve the phenomenology of a new class of hadronic states, which can be studied with nonrelativistic models with much more confidence than could the light-quark states. Our survey will concentrate on the latter aspects. By learning more about the bound states of heavy quarks we can learn more about properties of the quarks themselves, such as their charges and masses, and can predict new resonances and other phenomena that might signal the existence of still further quarks, such as the expected t quark mentioned in Sec. I.

Elementary mass relations

The $L=0$ hadrons containing a single charmed quark are shown in Fig. 27. They can be classified according to the $SU(3)$ representations of the remaining (light) quarks.⁸⁵ Charmed mesons then must belong to a $\bar{3}$ (their charge conjugates, to a 3). In charmed baryons, the light quarks can either be coupled to a $\bar{3}$ (with spin zero), leading to

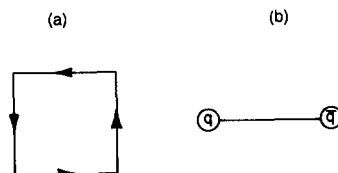


Fig. 26. Comparison of (a) simplest gluonic bound state and (b) simplest $q\bar{q}$ bound state in lattice gauge theory.

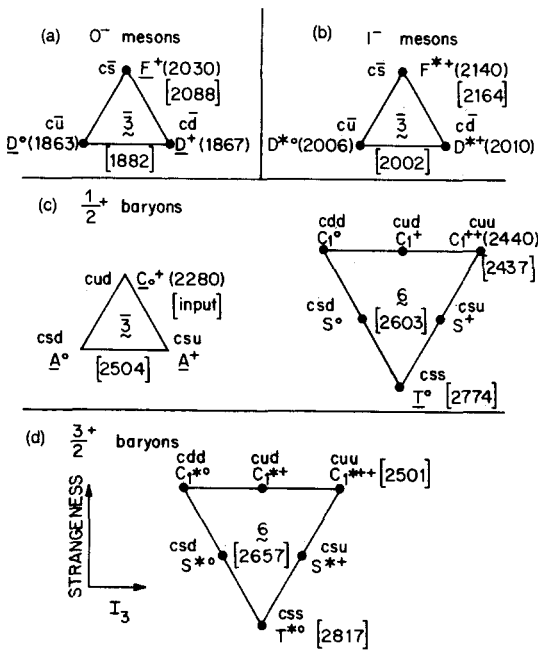


Fig. 27. $L = 0$ mesons and baryons containing a single charmed quark. $SU(3)$ representations are depicted in the middle of each figure. Masses in MeV/c^2 correspond to observed values (Ref.1 values in parentheses) or predicted values [(4.1), values in brackets]. Underlined states are expected to decay weakly.

states with spin $\frac{1}{2}$, or to a 6 (with spin 1), leading to states with spins $\frac{1}{2}$ and $\frac{3}{2}$. The model of Sec. III predicts

$$M_{\text{meson}}^{\text{(baryon)}} = \sum_i m_i + \left(\frac{a'}{a}\right) \sum_{i < j} \sigma_i \cdot \sigma_j / m_i m_j \quad (4.1)$$

for charmed states. All we need, in addition to the parameters determined earlier, is the charmed quark mass: this turns out to be $m_c = 1704 \text{ MeV}/c^2$ for baryons and $1662 \text{ MeV}/c^2$ for mesons. The predictions are shown as bracketed quantities in Fig. 27.

While this simple model is undoubtedly more crude than necessary, it suggests some interesting patterns whose validity could be more general.

(1) The difference between C_0^+ and A is due just to $m_c - m_s$ since the hyperfine interactions (between nonstrange quarks only) are the same for both. We can also form combinations of mesons for which the hyperfine interactions average to zero. These are just the spin-weighted combinations $(3D^* + D)/4$ and $(3K^* + K)/4$. (We denote a mass by its particle symbol, as before). Thus we expect

$$C_0^+ - A = (3D^* + D)/4 - (3K^* + K)/4, \quad (1164 \text{ MeV}/c^2) = (1179 \text{ MeV}/c^2), \quad (4.2)$$

which is not bad.

(2) The $D^* - D$ hyperfine splitting is predicted to be

$$D^* - D = (m_s/m_c)(K^* - K). \quad (4.3)$$

Indeed $D^* - D \approx \frac{1}{3}(K^* - K)$.

(3) The hyperfine interactions between nonstrange (u, d) and charmed or strange quarks vanish in the spin-weighted combinations $(2C_1^* + C_1)/3$ and $(2\Sigma^* + \Sigma)/3$. All that is left is a hyperfine interaction between nonstrange quarks, which is the same for both. Hence we expect that

$$(2C_1^* + C_1)/3 - C_0 = (2\Sigma^* + \Sigma)/3 - A \approx 205 \text{ MeV}/c^2.$$

(4) The $C_1^* - C_1$ and $\Sigma^* - \Sigma$ splittings are due to hyperfine

interactions of u or d quarks with the charmed or strange quark, so we expect⁸⁶

$$C_1^* - C_1 = \frac{m_s}{m_c} (\Sigma^* - \Sigma) = \frac{D^* - D}{K^* - K} (\Sigma^* - \Sigma) \approx 69 \text{ MeV}/c^2. \quad (4.5)$$

Combined with the prediction (4.4) this implies⁸⁷

$$C_1 - C_0 \approx 159 \text{ MeV}/c^2, \quad C_1^* - C_0 \approx 228 \text{ MeV}/c^2, \quad (4.6)$$

and both C_1 and C_1^* should be able to decay to $C_0 + \pi$. There does seem to be evidence for a $C_0\pi$ enhancement about $160 \text{ MeV}/c^2$ above C_0 .⁸⁸

A similar discussion applies, *mutatis mutandis*, to systems containing b quarks and u, d , or s quarks. We need one baryon mass or meson mass to get started. It looks as if $(b\bar{u}) + (b\bar{u})$ threshold lies somewhere between 10.35 and 10.58 GeV, since resonances at these two masses seem to be narrow and broader, respectively.⁸⁹ Let us estimate $M(b\bar{u}, 0^-) \approx 5.28 \text{ GeV}/c^2$. Let us also assume that $m_b - m_c$ is the same in a meson and a baryon, as is true for $m_c - m_s$ and $m_s - m_u$. Then we get the mass predictions of Fig. 28. The decay chains allowed by these masses are also shown. Numerous monochromatic photons should abound.

Adding up quarks is not enough for the J/ψ and Υ families. Binding energies and enhanced wave functions $|\Psi(0)|^2$ begin to be important when both quarks in the system are heavy. These effects are neglected in (4.1), which would give a J/ψ mass more than 200 MeV too high, an Υ mass more than 500 MeV too high, and a $^3S_1 - ^1S_0$ splitting of $\frac{1}{3}$ of the observed value in the ground state $c\bar{c}$ system. At this point it becomes necessary to introduce some dynamics.

Charmonium and Υ families: comparisons

The bound state of a quark and its antiquark can be called *quarkonium*, in analogy with positronium. Charmonium provides a rich spectroscopy of $c\bar{c}$ states, summarized in Fig. 29. In comparison with light-quark systems, charmonium has been an extremely clean system to study. Many of the states are below the charmed-pair threshold,

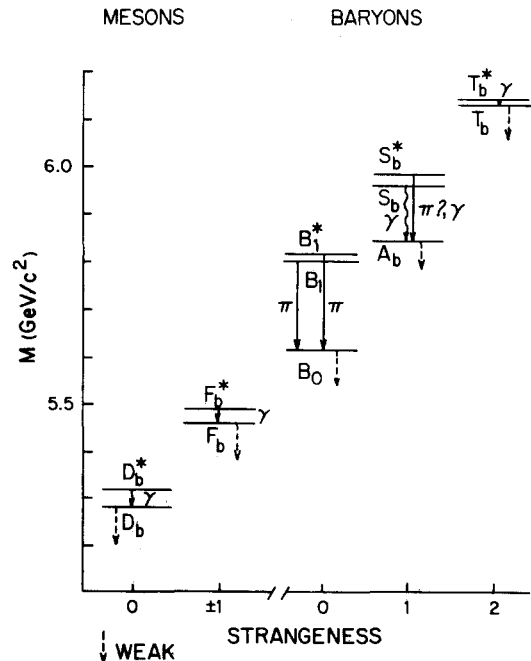


Fig. 28. $L = 0$ mesons and baryons containing a single b -flavored quark.

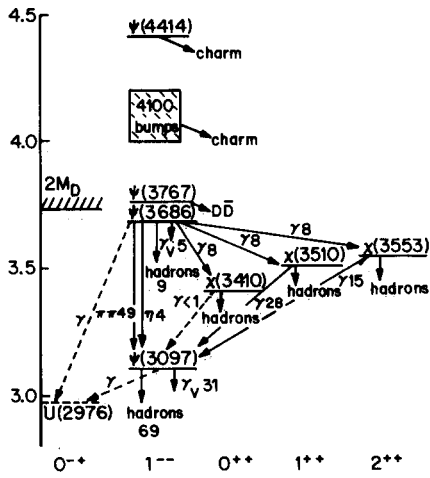


Fig. 29. Bound states of a charmed quark and charmed antiquark (charmonium). Decay modes and branching ratios (in percent) are shown.

and thus are extremely narrow in accord with the OZI (quark-line) rule mentioned in Sec. I. Several radial excitations (at least three above the ground state) have been seen. A great deal can be said about the quark-antiquark interaction as a result.

The charmonium and Υ families are compared in Fig. 30. Leptonic widths and masses are from Refs. 1, 89, 90, and 91. What do these systems tell us?

The masses provide information on the form of the potential $V(r)$ if we assume the validity of a nonrelativistic description in terms of the Schrödinger equation

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + V(r)\right)\Psi(r) = E\Psi(r). \quad (4.7)$$

Here $\mu = m/2$ is the reduced mass, m is the quark mass, and the mass M of the bound state is the energy eigenvalue E plus $2m$. The leptonic widths are related to the square of the S -wave eigenfunction at the origin, $|\Psi(0)|^2$, by the expression⁹²

$$\Gamma(V \rightarrow e^+e^-) = \frac{16\pi\alpha^2 e_Q^2}{M_V^2} |\Psi(0)|^2, \quad (4.8)$$

where V denotes a vector (spin-1) meson, M_V its mass, and e_Q is the quark charge. It is surprising how much can be learned about $V(r)$ from the data. Several approaches have been taken:

1. *Explicit potentials.* One can construct potentials incorporating theoretical prejudices about short- and long-range behavior and compare them with data. A simple example is

$$V(r) = -\frac{4}{3}\alpha_s/r + kr. \quad (4.9)$$

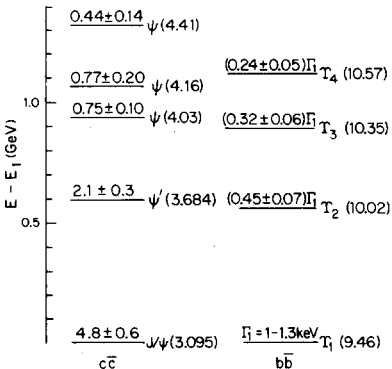


Fig. 30. Comparison of charmonium ($c\bar{c}$) and Υ ($b\bar{b}$) levels. Leptonic widths in keV are shown above the corresponding level.

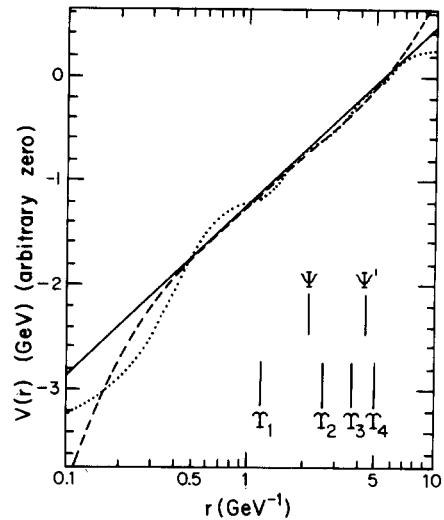


Fig. 31. Three potentials $V(r)$ that fit the charmonium and upsilon data. --- QCD inspired (Buchmüller-Tye⁹³); ---- inverse scattering method⁹⁴; - $V(r) = (0.73 \text{ GeV}) \log r$.

A more refined version takes into account asymptotic freedom at short distances⁹³

$$\tilde{V}(\mathbf{q}^2) \equiv \int d^3r e^{-i\mathbf{q}\cdot\mathbf{r}} V(r) = \frac{4}{3} \frac{\hat{\alpha}_s(\mathbf{q}^2)}{q^2} \quad (4.10)$$

with

$$1/\hat{\alpha}_s(\mathbf{q}^2) = \left[(11 - \frac{2}{3}N_f)/4\pi \right] \log(1 + \mathbf{q}^2/\Lambda^2). \quad (4.11)$$

The 1 in the logarithm is a phenomenological insertion that guarantees that $V(q^2) \sim (q^2)^{-2}$ as $q^2 \rightarrow 0$, and hence $V(r) \sim r$ at long distances.

2. *Scaling laws.* Simple potentials without theoretical motivation are useful mnemonics for recalling many features of the data simultaneously.⁹¹ An example is

$$V(r) = \lambda r^\nu. \quad (4.12)$$

It appears that a small power $\nu \approx 0.94$ describes level spacings and leptonic widths of the charmonium and upsilon levels. [The nontrivial limit of (4.12) for $\nu \rightarrow 0$ is obtained by letting $\lambda \nu \rightarrow \text{const.}$ and subtracting an infinite constant. The result is $V(r) = \text{const.} \log(r)$.]

3. *Construction of potentials from data.* It is possible to determine a potential in the Schrödinger equation (4.7) if certain scattering and bound-state data are known. Under some circumstances, bound-state data alone suffice. This "inverse scattering" program has been applied to quarkonium⁹⁵ and we shall mention some of its results presently.

Corresponding to the above three approaches, three types of potential, all of which fit charmonium and upsilon data, are compared in Fig. 31. In the region probed by

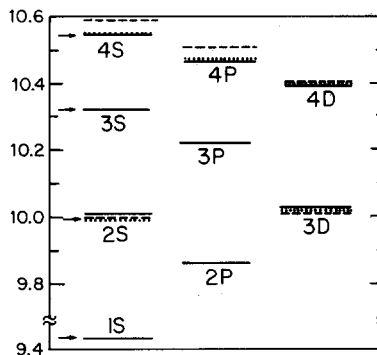


Fig. 32. Predictions of Υ system levels for three potentials ---- Buchmüller-Tye⁹³; inverse scattering⁹⁴; - $V(r) = (0.715 \text{ GeV}) \log r$.

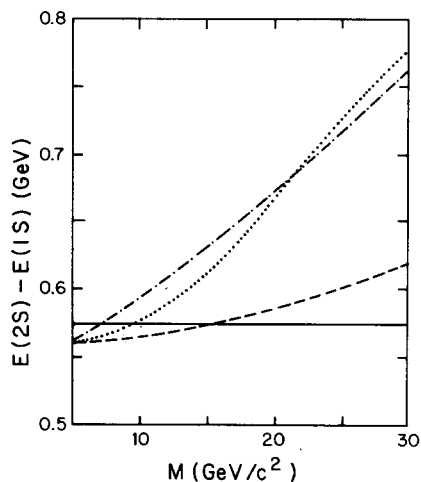


Fig. 33. 2S-1S level spacing as a function of the quark mass for four potentials: --- QCD inspired ($\Lambda = 200$ MeV); - - - QCD inspired ($\Lambda = 500$ MeV) (Buchmüller-Tye⁹³); inverse scattering⁹⁴; - · - · - $V(r) = (0.715 \text{ GeV}) \log r$.

present data, they are remarkably similar, as are the predictions for other Υ levels shown in Fig. 32. The lowest P -wave level (which we call $2P$ in analogy with the hydrogen spectrum) is closer to the $2S$ level for potentials more singular near the origin. Indeed, in potentials of the form (4.12), we expect

$$\frac{2P - 1S}{2S - 1S} = \begin{cases} \frac{1}{2} & \nu = 2 \\ \simeq \frac{1}{4} & \nu = 0 \\ 0 & \nu = -1 \end{cases} \quad (4.13)$$

To learn more about the quark-antiquark potential at short distances we have to resort to heavier quarks. Let us study just two examples of how a heavy $t\bar{t}$ system could be of use in this context. Many more illustrations have been given in recent reviews.

1. *Level spacings.* In the Schrödinger equation (4.7) with the potential (4.12), it is simple to absorb a power of the quark mass into the coordinate. In this manner one learns that the level spacings ΔE scale with m as

$$\Delta E \sim m^{-\nu/(2+\nu)} \quad (4.14)$$

They are independent of m for $\nu = 0$. The similarity of the charmonium and upsilon levels in Fig. 30 is a strong reason why the data suggests $\nu \simeq 0$. For $\nu = 0$ ($V \sim \log r$), the potential in Fig. 31 is a straight line.

Heavier quarks probe shorter distances because of their

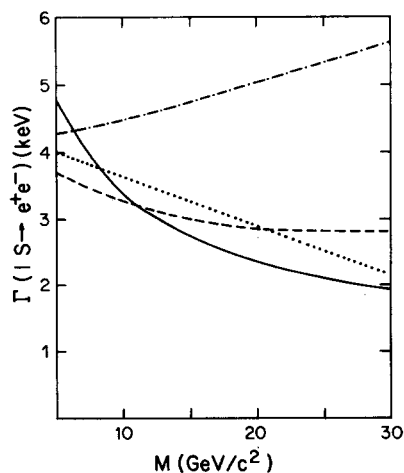


Fig. 34. Leptonic decay rate $\Gamma(1S \rightarrow e^+e^-)$ keV, as a function of quark mass for four potentials; - - - Buchmüller-Tye⁹³ ($\Lambda = 200$ MeV); - · - · - Buchmüller-Tye⁹³ ($\Lambda = 500$ MeV); inverse scattering⁹⁴; - · - · - $V(r) = (0.715 \text{ GeV}) \log r$.

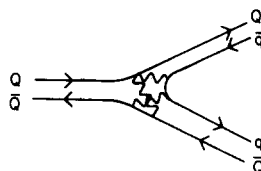


Fig. 35. Mechanism for decay of quarkonium above flavor threshold into mesons carrying flavor of heavy quarks Q .

smaller Compton wavelength. If theory is any guide, the effective power of the potential should decrease below $\nu = 0$ at short distances, as shown by the curvature in Fig. 31. The level spacings should then increase with m . We show in Fig. 33 some expectations for the 2S-1S spacing in various potentials as a function of m .

2. *Leptonic widths.* The shortest-distance information at any quark mass is provided by the leptonic width of the lowest 1S level. Scaling arguments for potentials (4.12) imply that

$$|\Psi(0)|^2 \sim m^{3/(2+\nu)} \quad (4.15)$$

The smaller ν is, the more rapidly $|\Psi(0)|^2$ grows with m . The predictions of several potentials for the 1S and 2S leptonic widths are compared in Fig. 34 as functions of m .

Semiclassical techniques

Some useful predictions of potential models may be made with the help of the WKB approximation.⁹¹ For S -wave levels in a central potential, the principal quantum number and the energy are related by

$$\int_0^{r_0} dr p(r) = (n - \frac{1}{4})\pi\hbar, \quad (4.16)$$

where

$$p(r) \equiv \{2\mu[E - V(r)]\}^{1/2} \quad V(r_0) = E. \quad (4.17)$$

The $-\frac{1}{4}$ in (4.16) is appropriate for nonsingular potentials. It is slightly modified for singular ones: for $V \sim r^\nu$ ($\nu < 0$) near $r = 0$ it is $-(1 + \nu)/[2(2 + \nu)]$ instead. For large n one then finds by scaling arguments that with a potential (4.12) the energy levels behave as

$$\Delta E \sim n^{2\nu/(2+\nu)} \quad (4.18)$$

As long as $\nu < 2$ the levels get closer together as n increases. This behavior is clearly visible in Fig. 30. The Υ spectrum in Fig. 30 actually favors a value of ν very close to 0.

An elementary manipulation of the radial Schrödinger equation yields the useful relation

$$|\Psi(0)|^2 = \frac{\mu}{2\pi} \left\langle \frac{dV}{dr} \right\rangle. \quad (4.19)$$

With the help of WKB relations such as (4.16) (a slight generalization is needed for singular potentials), one finds for large n ,

$$|\Psi(0)|^2 \sim \begin{cases} n^{2(\nu-1)/(\nu+2)} & \nu \geq 0 \\ n^{(\nu-2)/(\nu+2)} & \nu < 0. \end{cases} \quad (4.20)$$

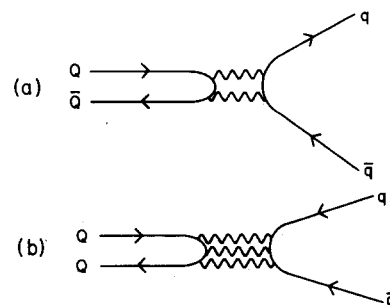


Fig. 36. OZI³⁰ suppressed decays of quarkonium below flavor threshold: (a) for $C = + Q\bar{Q}$ states; (b) for $C = - Q\bar{Q}$ states.

(More refined results are presented in Ref. 91.) The observed values of $|\Psi(0)|^2$ for charmonium and Υ families behave roughly as $1/n$, again supporting $v \approx 0$.

Flavor thresholds

One amusing and model-independent prediction can be made from the WKB expression (4.16). This concerns the energy at which the quark and antiquark in quarkonium can dissociate from one another and appear in separate hadrons, each of which contains a single heavy quark. Quarkonium states above this energy can be broad (e.g., \approx several tens of MeV) as a result of the high decay probability for this process (Fig. 35). Below this energy ("flavor threshold") quarkonium must decay via $q\bar{q}$ annihilation to gluons, which then couple to light quarks (Fig. 36). A colorless $q\bar{q}$ state cannot couple to a single (color octet) gluon. A $C = -$ color singlet state such as a 3S_1 meson produced in e^+e^- annihilation cannot couple to two gluons either. Hence at least three gluons must be emitted by a 3S_1 quarkonium state below flavor threshold, leading to a suppression of the decay rate and a corresponding narrowing (typically to several tens of keV) of the resonance. The gluon-quark coupling is of the order of magnitude 0.2–0.4 (for α_s) in the charmonium range⁷⁵ and the expectation is that it is smaller for Υ decays. This is the explanation for the unexpected success of the OZI suppression rules³⁰ for quarkonia, and the spectacular signals in e^+e^- annihilations of resonances like the ψ , ψ' , and lowest Υ levels.

The WKB prediction for flavor threshold E_{th} starts from the observation that the energy scales both for quarkonium levels and for flavored pairs are set by $2m$.⁹⁶ Thus while $M = E + 2m$, we expect $E_{th} = \delta + 2m$, where δ is a few hundred MeV and reflects the dynamics whereby a light quark is bound to a heavy one. The quantity δ should depend on m , but only through reduced mass and hyperfine effects. As $m \rightarrow \infty$, we expect $\delta \rightarrow \delta_\infty$. The value of n corresponding to flavor threshold should then be [neglecting the small negative constant in (4.16)]

$$n_{th} \cong \frac{2}{h} \int_0^{r_0} dr p(r) \quad (4.21)$$

or

$$n_{th} \cong \text{const} \sqrt{m}. \quad (4.22)$$

We may evaluate the constant for charmonium

$$n_{th} \cong 2(m/1.6 \text{ GeV})^{1/2}, \quad (4.23)$$

since flavor threshold is just above the second 3S_1 level, or from $b\bar{b}$ systems,

$$n_{th} \cong (3 - 4)(m/5 \text{ GeV})^{1/2}. \quad (4.24)$$

The flavor threshold is known in the Υ family to lie between the third (narrow) and the fourth (broad) state. It will be better known when B -flavored mesons are discovered.

Quark charges

So far, in applying the expression (4.7), we have assumed Γ and e_Q^2 are known, and extract $|\Psi(0)|^2$. We can use the expression to learn e_Q^2 instead, if $|\Psi(0)|^2$ is well-enough known. To this end we may use (4.19) to establish a theorem.⁹⁷

If

$$V' > 0, \quad V'' < 0,$$

then

$$\frac{\partial}{\partial m} \left(\frac{1}{m} |\Psi(0)|^2 \right) > 0 \quad (4.25)$$

for the ground state. The conditions of this theorem are satisfied by potentials such as (4.9), (4.10), or (4.12) for $v < 1$. It appears that the results hold approximately for higher states as well, though this has not been established rigorously.

The value of $|\Psi(0)|^2$ for the ground state of a high-mass $q\bar{q}$ state is new information not provided by any lower-mass quarkonium system. By contrast, $|\Psi(0)|^2$ for excited states probes [via (4.19)] parts of the potential already sampled by the lowest levels of lower-mass quarkonia. This may be seen by comparison of mean radii of ψ and Υ levels shown in Fig. 31. As a result, one may be able to predict $|\Psi(0)|^2$ for excited states with more certainty. By using (4.8) and experimental leptonic widths, it is then possible to distinguish among various possibilities for the quark charge. This method gave an early indication, on the basis of the Υ' leptonic width, that the b quark has charge $-\frac{1}{3}$.

Gluon and photon emission

A quarkonium system can decay via annihilation into two or three gluons or photons, depending on whether $C = +$ or $-$. In Table XII we give rates for these processes for 1S_0 and 3S_1 states. Other expressions (for P states, etc.) can be found in Ref. 98. The hadronic decays of J/ψ may be compared with the leptonic ones [given by (4.8)] to obtain a value for α_s :

$$\frac{\Gamma(J/\psi \rightarrow \text{hadrons})}{\Gamma(J/\psi \rightarrow l^+l^-)} = \frac{45 \pm 12 \text{ keV}}{4.8 \pm 0.6 \text{ keV}}, \quad (4.26)$$

yielding $\alpha_s(J/\psi) \approx 0.19$. There are QCD radiative corrections that affect both the hadronic and leptonic decay processes; so far they have only been calculated for the latter. A similar computation for the Υ yields $\alpha_s(\Upsilon) \approx 0.17$. This is close to the value extracted from J/ψ . It is too uncertain to detect the expected decrease at higher masses that is expected from asymptotic freedom.¹⁰⁷

One interesting difficulty of the quark description of charmonium is the apparent suppression of decay rates for the process $\psi' \rightarrow \gamma + \chi$. This should be an electric dipole transition. (The χ states are shown in Fig. 29.) Simple non-relativistic calculations^{74,98} yield rates about three times as large as those observed. Does this mean that there is an important nonquark component in the charmonium wave function, or only that relativistic corrections are still important for charmonium? This last suggestion is not unreasonable, since comparison of the observed level spacing in

Table XII. Decay rates for quarkonium S states into photons and gluons. ^a

Decay	Rate
$^1S_0 \rightarrow 2\gamma$	$4\pi\alpha^2 [\Psi(0) /m_q^2] (\sum C_i e_{Q_i}^2)^2$
$^3S_1 \rightarrow 3\gamma$	$[16(\pi^2 - 9)\alpha^3/3] [\Psi(0) ^2/m_q^2] (\sum C_i e_{Q_i}^3)^2$
$^1S_0 \rightarrow 2g$	$[8\pi\alpha_s^2/3] [\Psi(0) ^2/m_q^2]$
$^3S_1 \rightarrow 3g$	$[40(\pi^2 - 9)\alpha_s^3/81] [\Psi(0) ^2/m_q^2]$

^a The initial state is taken as a linear combination $|\text{init}\rangle = \sum_i c_i |q_i \bar{q}_i\rangle$, where the sum is over both colors and flavors. For a single flavor $c_1 = c_2 = c_3 = 1/\sqrt{3}$.

charmonium with the quark mass yields an estimate for $v/c \cong \frac{1}{2}$. The rates for $\Upsilon' \rightarrow \gamma + \chi_b$ will be very helpful in solving this dilemma. A nonrelativistic calculation predicts branching ratios of several percent for each of the processes $\Upsilon' \rightarrow \gamma \chi_b$ ($J = 2, 1, 0$).

In summary, it is fair to say, that once the shock of the discovery of quarkonia has worn off, the application of nonrelativistic quantum mechanics with guidance from QCD has led to a remarkable degree of understanding of the spectra and decays of these states.

V. THE NEXT STEP: GRAND UNIFICATION

The theories of the strong and electroweak interactions of the quarks and leptons are still in the process of being worked out. Many difficult problems remain to be solved, and many experimental checks that will take years to carry out remain. In spite of this there is unprecedented optimism in the community of high-energy physicists, because for the first time there are real theories for these interactions, theories that, in spite of their variety, share a common basic principle: they are all consequences of underlying *local symmetries*. The conviction that all the difficult problems will be solved sooner or later, and that the present qualitative correctness of the theories will in time become quantitative, has led a number of theoretical physicists to suggest that the current picture of

$$SU(3)_{\text{color}} \times SU(2)_{\text{electroweak}} \times U(1) \xrightarrow[\text{breaking}]{\text{spontaneous}} SU(3)_{\text{color}} \times U(1)_{\text{e.m.}}$$

is the last step of a chain of symmetry loss from a deeper symmetry that unifies all of the observed interactions. Could the present theories that describe (one hopes) phenomena on a scale $\lesssim 100\text{--}1000$ GeV be "low-energy" manifestations of an even more symmetric theory? Given the range of energies in which experiments are (and will be) possible, such a question would be absurd, except that reliance on the gauge principle allows for organized speculation. The speculations that will be discussed here stop short of the inclusion of gravitation, because the quantum theory of gravitation is not yet understood. Fortunately, this leaves a great deal of room. The energy scale at which gravitational interactions become strong may be estimated on dimensional grounds to be

$$E \sim (\hbar c/G)^{1/2} c^2 \sim 10^{19} \text{ GeV}, \quad (5.1)$$

and the grand unified theories (GUT) under consideration operate at energies that are low compared to the Planck mass $(\hbar c/G)^{1/2}$ in (5.1). What one is looking for is a local symmetry under some group G that has as its subgroups $SU(3) \times SU(2) \times U(1)$. Because there is as yet no clear understanding of how bound states are formed, and what the constraints on these are, the conservative attitude that G operates on the "familiar" quarks and leptons (rather than some subunits that make up these particles) has been adopted. Since it is not known how many more quarks and leptons really exist, present speculations are restricted to dealing with *generations* of leptons and quarks that appear to be the appropriate units at the "low" energies. There are suggestions that the generations are

$$\text{I} \quad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \quad e_R^- \quad u_R \quad d_R$$

$$\text{II} \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \mu_R^- \quad c_R \quad s_R \quad (5.2)$$

$$\text{III} \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L \quad \tau_R^- \quad t_R \quad b_R$$

with each quark coming in three colors. There exists for spin- $\frac{1}{2}$ particles a relationship between chirality and charge conjugation, and it is more elegant to replace all the right-handed spinors by the charge conjugate left-handed ones. Thus the first generation, for example, can be written as

$$\psi_{e, e^-, u, d, \bar{u}, \bar{d}} \quad i = 1, 2, 3 \text{ (color)}, \quad (5.3)$$

giving a total of 15 two-component fields (16, if the neutrino had a mass).

There are conditions that are imposed on the possible group G . These are

(i) The group should be simple, so that there is only one coupling constant. An immediate bonus is that the charges of the leptons and quarks must be simply related, and the mystery why the electron and proton charges are equal

$$|(Q_e + Q_p)/Q_p| < O(10^{-21}) \quad (5.4)$$

may be solved.

(ii) The group G must contain $SU(3) \times SU(2) \times U(1)$. Since these have rank 2, 1, 1, respectively, G must have rank 4 at least. A restriction to rank 4 narrows the choice enormously.

(iii) Because the families (5.3) involve fields and their conjugates, representations that are complex must exist in G . The analysis of H. Georgi and S. L. Glashow⁹⁹ restricts the choice to a *single group*, $SU(5)$.

The fundamental representations of $SU(5)$ are the **5** and $\bar{\mathbf{5}}$. Another low-multiplicity representation is the **10** arising from the antisymmetric combination of $\mathbf{5} \times \mathbf{5}$. Interestingly enough, each family can be distributed into the $\bar{\mathbf{5}} \oplus \mathbf{10}$ as follows:

$$\bar{\mathbf{5}} = \begin{bmatrix} \bar{d}_R \\ \bar{d}_Y \\ \bar{d}_B \\ e^- \\ \nu_e \end{bmatrix}_L, \quad \mathbf{10} = \begin{bmatrix} 0 & \bar{u}_B & -\bar{u}_Y & -u_R & -d_R \\ -\bar{u}_B & 0 & \bar{u}_R & -u_Y & -d_Y \\ \bar{u}_Y & -\bar{u}_R & 0 & -u_B & -d_B \\ u_R & u_Y & u_B & 0 & -\bar{e} \\ d_R & d_Y & d_B & \bar{e} & 0 \end{bmatrix}_L, \quad (5.5)$$

where we have used the color indices R, Y, B . To see how these come about, we note that in terms of the transformation properties under $SU(3)$ and $SU(2)$ the **5** representation of $SU(5)$ decomposes as follows

$$\mathbf{5}: (\mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{2}), \quad (5.6)$$

that is, a color triplet that is a singlet under $SU(2)$, and a color singlet that is a doublet under $SU(2)$. Thus in the multiplication of $\mathbf{5} \oplus \mathbf{5}$ we have

$$\begin{aligned} [(\mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{2})] \otimes [(\mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{2})] &= ((\mathbf{3}, \mathbf{1}) \otimes (\mathbf{3}, \mathbf{1})) \\ &\oplus ((\mathbf{3}, \mathbf{1}) \otimes (\mathbf{1}, \mathbf{2})) \oplus ((\mathbf{1}, \mathbf{2}) \otimes (\mathbf{3}, \mathbf{1})) \oplus ((\mathbf{1}, \mathbf{2}) \otimes (\mathbf{1}, \mathbf{2})) \\ &= (\mathbf{3}, \mathbf{1}) \otimes (\mathbf{3}, \mathbf{2})_A \oplus (\mathbf{1}, \mathbf{1}) \otimes (\mathbf{6}, \mathbf{1}) \\ &\quad \oplus (\mathbf{3}, \mathbf{2})_S \oplus (\mathbf{1}, \mathbf{3}). \quad (5.7) \end{aligned}$$

We used the fact that under $SU(3)$, $3 \otimes 3 = 6 \oplus \bar{3}$, and under $SU(2)$, $2 \otimes 2 = 3 \oplus 1$, and rearranged things so that the first line contains the antisymmetric part of the product, and the second line the symmetric part. If we try to identify parts of the 10 with parts of the family in (5.3) we see that

$$(\bar{3}, 1) = \bar{u} \text{ or } \bar{d}; \quad (3, 2) = \begin{pmatrix} u \\ d \end{pmatrix}; \quad (1, 1) = \bar{e}. \quad (5.8)$$

Since either a \bar{d} or a \bar{u} must be added to this, we can take

$$(\bar{3}, 1) + (1, 2) = \bar{5} = \bar{d} \text{ or } \bar{u} + \begin{pmatrix} \nu_e \\ e \end{pmatrix}. \quad (5.9)$$

The choice in favor of \bar{u} in the 10 is guided by consideration of the properties under $U(1)$ that have not been considered explicitly. Since the electric charge is one of the generators, and thus represented by a traceless (diagonal) matrix acting on the 5 or $\bar{5}$, the sum of the electric charges in a multiplet must vanish. Thus if the \bar{d} is put in the $\bar{5}$ together with the left-over $\begin{pmatrix} \nu_e \\ e \end{pmatrix}$ we get, because of the three colors

$$-3Q_d + Q_e = 0. \quad (5.10)$$

Thus the fundamental relation (5.4) is explained. It is interesting that fractional charges were introduced in connection with the three flavors required by Gell-Mann and Zweig to account for the hadronic states known in the early 1960s; in actuality it appears that the fractional charge is related to color.

It is an important technical consequence of the tracelessness of the charge operator and the choice of the representation $\bar{5} + 10$ that the triangle anomalies mentioned in Sec. I cancel in each generation, so that renormalizability can be maintained.

The appearance of quarks and leptons in the same representation means that under the operation of the symmetry group they can convert into each other, with the emission of one of the $5^2 - 1 = 24$ gauge vector bosons. Of the 24 gauge vector bosons, $8 + 3 + 1$ are already familiar to us: they are the massless QCD gluons, the massive (by virtue of the spontaneous symmetry breakdown), W^\pm and Z^0 , and the photon. The remaining 12 have lepto-quark attributes, and if the theory is not to violate existing evidence for baryon conservation, the transition rates that they induce must lead to a proton lifetime in excess of the currently observed limit of $\sim 10^{30}$ yr.² This cannot be achieved by vanishingly small coupling since there is only one coupling constant; it can only come about for the same reason that the weak interactions manifest themselves as so much weaker than the electromagnetic ones: the vector bosons must be very massive. Since the effective coupling is of the order of g^2/M_X^2 , where M_X is the mass of the vector boson, the typical estimate of the proton lifetime, using dimensional arguments, is

$$\tau_p = (\text{const.})(1/\alpha_G^2)(M_X^4/m_p^5). \quad (5.11)$$

We shall estimate α_G^2 to be of order 10^{-3} , and this means that M_X must be larger than $\sim 10^{15}$ GeV/ c^2 . Thus there must be a spontaneous symmetry breakdown of $SU(5)$ that occurs at energies of this magnitude (which is still small compared with the Planck mass of 10^{19} GeV/ c^2). Such a breakdown leaves the $SU(2)$ masses as well as those of the $SU(3)$ gluons and the $U(1)$ "photon" zero. The $SU(2)$ masses leave that status at the second stage of symmetry breaking at a scale of the order of 100 GeV, and become the W^\pm and Z^0 . The 10^{15} -GeV/ c^2 scale is actually more easily understood than the 100-GeV/ c^2 scale. At a scale set by M_X all the couplings are equal, but since the coupling constants

are "running" coupling constants that depend on Q^2 (as we saw in Sec. III), the couplings g_3 and g_2 increase for lower energies (asymptotic freedom), while g_1 for the Abelian group $U(1)$ decreases to the observed value at hadronic energies. One may use the logarithmic dependence of the running coupling constants on M_X^2/Q^2 to estimate the magnitude of M_X (the dependence is schematically shown in Fig. 37).

To estimate M_X we note that

$$\alpha_s(Q^2) \equiv g_3^2(Q^2)/4\pi, \\ \alpha_2(Q^2) \equiv g_2^2(Q^2)/4\pi = g^2(Q^2)/4\pi \quad (5.12)$$

obey (2.8), which for $SU(3)$ and $SU(2)$, respectively, take the form

$$\frac{1}{\alpha_s(Q^2)} - \frac{1}{\alpha_s(M_X^2)} = -\frac{1}{4\pi} (11 - \frac{2}{3}N_f) \log \frac{M_X^2}{Q^2}, \quad (5.13)$$

$$\frac{1}{\alpha_2(Q^2)} - \frac{1}{\alpha_2(M_X^2)} = -\frac{1}{4\pi} (\frac{22}{3} - \frac{2}{3}N_f) \log \frac{M_X^2}{Q^2}. \quad (5.14)$$

Here N_f is the number of flavors, which plays little role below. For the $U(1)$ coupling,

$$\frac{1}{\alpha_1(Q^2)} - \frac{1}{\alpha_1(M_X^2)} = \frac{1}{4\pi} (\frac{4}{3}N_f) \log \frac{M_X^2}{Q^2}. \quad (5.15)$$

At the unification mass,

$$\alpha_1(M_X^2) = \alpha_2(M_X^2) = \alpha_s(M_X^2). \quad (5.16)$$

We digress briefly for a small technical point: in the Weinberg-Salam theory the couplings are schematically written as

$$gT_3 + \frac{1}{2}g'Y, \quad (5.17)$$

and the hypercharge Y is related to the electric charge Q by

$$Q = T_3 + \frac{1}{2}Y. \quad (5.18)$$

T_3 and Y are both generators of $SU(5)$, together with the remaining 22 generators, and they must all be normalized in the same way before one can append to them a universal coupling constant. To do this in the most economical way we shall carry out the normalization by evaluating

$$\sum_{\text{rep}} Q^2 = \sum_{\text{rep}} (T_3 + \frac{1}{2}Y)^2 = \sum_{\text{rep}} [T_3 + (c/2)T_Y]^2 \quad (5.19)$$

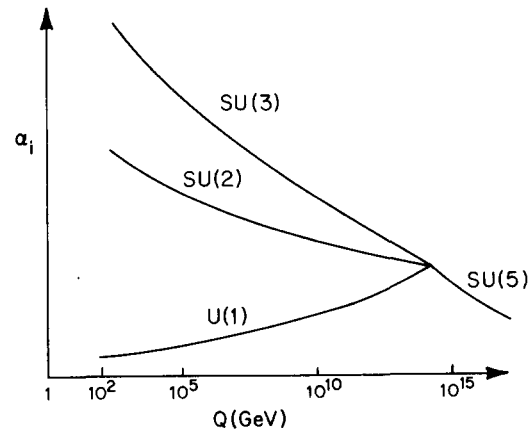


Fig. 37. Evolution of $SU(5)$ coupling constant into subgroup couplings for $SU(3)$, $SU(2)$ (increasing as Q decreases by asymptotic freedom), and $U(1)$ (decreasing, as for quantum electrodynamics, with decreasing Q).

for the representation 5 in (5.5). We have

$$\begin{aligned} \sum_{\text{rep}} Q^2 &= 3\left(\frac{1}{3}\right)^2 + 1 = \frac{4}{3}, \\ \sum_{\text{rep}} T_3^2 &= \left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 = \frac{1}{2}, \\ c \sum_{\text{rep}} T_3 T_Y &= \sum_{\text{rep}} T_3 Y = (-1)\left[\frac{1}{2} + \left(-\frac{1}{2}\right)\right] = 0, \\ c^2 \sum_{\text{rep}} T_Y^2 &= \frac{1}{3} c^2 \sum_{\text{rep}} T_3^2 = c^2/8, \\ \therefore \frac{4}{3} &= \frac{1}{2} + c^2/8, \end{aligned} \quad (5.20)$$

and hence the relationship between Y and the properly normalized T_Y is given by

$$Y = 2\sqrt{\frac{3}{5}} T_Y. \quad (5.21)$$

Thus the coupling in the Weinberg-Salam theory is

$$g T_3 + g' \sqrt{\frac{3}{5}} T_Y \equiv g_2 T_3 + g_1 T_Y. \quad (5.22)$$

This implies that for the Weinberg angle θ_w ,

$$\sin^2 \theta_w = g'^2 / (g^2 + g'^2) = \frac{3}{5} g_1^2 / (g_2^2 + \frac{3}{5} g_1^2), \quad (5.23)$$

and in the unification limit this takes the value $\frac{3}{8}$. At hadronic energies, g_1 decreases and g_2 increases¹⁰⁰ and the angle is closer to the experimental value. A relation that follows from (1.25)–(1.27) is

$$e^2 = g^2 g'^2 / (g^2 + g'^2) = \frac{3}{5} g_2^2 g_1^2 / (g_2^2 + \frac{3}{5} g_1^2) \quad (5.24)$$

or equivalently

$$\alpha = \frac{3}{5} \alpha_2 \alpha_1 / (\alpha_2 + \frac{3}{5} \alpha_1) = \alpha_2 \sin^2 \theta_w. \quad (5.25)$$

Subtraction of (5.14) from (5.15) leads, after some algebra, to

$$\sin^2 \theta_w = \frac{3}{8} \left[1 - (\alpha/4\pi) \frac{110}{9} \log(M_X^2/Q^2) \right], \quad (5.26)$$

and similar manipulations lead to

$$\alpha/\alpha_2(Q^2) = \frac{3}{8} \left\{ 1 - (\alpha/2\pi) [11 \log(M_X^2/Q^2)] \right\}. \quad (5.27)$$

When reasonable values of α_s are used [$A = 300$ MeV and $N_f = 4$ below the heavy quark thresholds, in (2.9)], one obtains¹⁰¹

$$M_X = 3.7 \times 10^{16} \text{ GeV}/c^2, \quad \sin^2 \theta_w(M_X^2) = 0.20. \quad (5.28)$$

More careful calculations that include (i) the Q^2 dependence of α , taken to be 1/137 above; (ii) the effects of thresholds; (iii) higher-order corrections to the renormalization group equations; and (iv) the inclusion of the effects of the Higgs boson, all reduce M_X , with a total reduction of nearly 100. The net effect of all of this is to yield the predictions^{17,102,103}

$$\sin^2 \theta_w = 0.209 \pm 0.005, \quad (5.29)$$

to be compared with the experimental value of 0.215 ± 0.012 . The proton lifetime is estimated to be

$$\tau_p \simeq 10^{31 \pm 2} \text{ yr}, \quad (5.30)$$

which puts it in a measurable range.

The arguments outlined above, and especially the calculation of $\sin^2 \theta_w$, strongly suggest that $SU(5)$ is a good candidate for the next stage in the unification of the interactions. This is not the last word in unification, since $SU(5)$ does not explain why there are three generations or how these are related to each other, nor does it explain how the pattern of symmetry breakings occurs. Nevertheless, the notion of a unifying group fits so naturally into the gauge principle framework that the prediction of baryon instabil-

ity must be taken very seriously. It is expected that within a few years the prediction (5.30) can be checked.

CONCLUSION

The quark model began as little more than a quantum-number counting device. After a brief period during which quarks only played a symmetry role, serious interest in quark dynamics developed. The marriage of the principle of local gauge invariance and quarks has been astonishingly productive. Although many questions still need to be answered, there is little doubt that the strong, weak, and electroweak interactions of matter are described by gauge theories of interactions of the quarks. Our review has focussed on the successes. Whatever difficulties there are—and it may be many years before they are overcome, and reliable calculations from first principles become possible—they should not blind us to the fact that in the last 20 years the field of elementary particle physics has been illuminated in an almost unprecedented way.

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FOOTNOTES

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¹⁰⁶A necessary condition for renormalizability is that all parameters that appear in the Lagrangian have dimensions of (mass) to a power that is ≥ 0 . The dimensions of the fields are determined by the requirement that the action, the integral of the Lagrangian density over space-time, is dimensionless ($\hbar = c = 1$).

¹⁰⁷The radiative corrections to the hadronic process have now been calculated by P. Mackenzie and G. P. Lepage (unpublished), and tend to reduce slightly the value of α_s extracted from (4.26). Thus for \mathcal{T} , one now finds $\alpha_s \approx 0.15$.

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